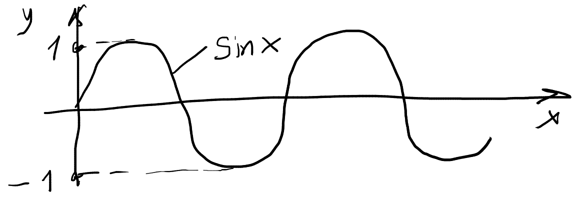
10. TRIGONOMETRIJSKE FUNKCIJE

1 (Pr 3)

$$\sin x = \frac{2a+1}{a-2}$$



$$-1 \leq \sin x \leq 1$$

I)
$$\frac{2a+1}{a-2} \geq -1$$

$$\frac{2a+1}{a-2} + 1 \geq 0$$

$$\frac{2a+1+(a-2)}{a-2} \geq 0$$

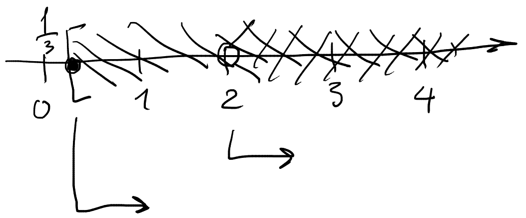
$$\frac{3a-1}{a-2} \geq 0 \qquad \left[\frac{x}{0}, \frac{0}{0}\right] ;\ a-2 > 0$$

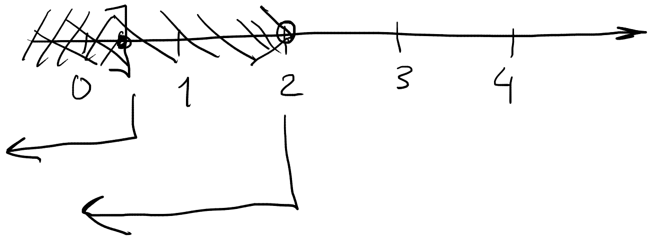
1) $+$ $3a-1 \geq 0 \Rightarrow a \geq \frac{1}{3}$

$+$ $a-2 > 0 \Rightarrow a > 2$

2) $-$ $3a-1 \leq 0 \Rightarrow a \leq \frac{1}{3}$

$-$ $a-2 < 0 \Rightarrow a < 2$





$$a_I \in \left\langle -\infty, \frac{1}{3}\right] \cup \langle 2, +\infty \rangle$$

II)
$$\frac{2a+1}{a-2} \leq 1$$

$$\frac{2a+1}{a-2} - 1 \leq 0$$

$$\frac{2a+1-(a-2)}{a-2} \leq 0$$

$$\frac{a+3}{a-2} \leq 0$$

1) $-$ $a+3 \leq 0 \Rightarrow a \leq -3$

$+$ $a-2 > 0 \Rightarrow a > 2$

2) $+$ $a+3 \geq 0 \Rightarrow a \geq -3$

$-$ $a-2 < 0 \Rightarrow a < 2$

$$\to a_{II} \in [-3, 2\rangle$$

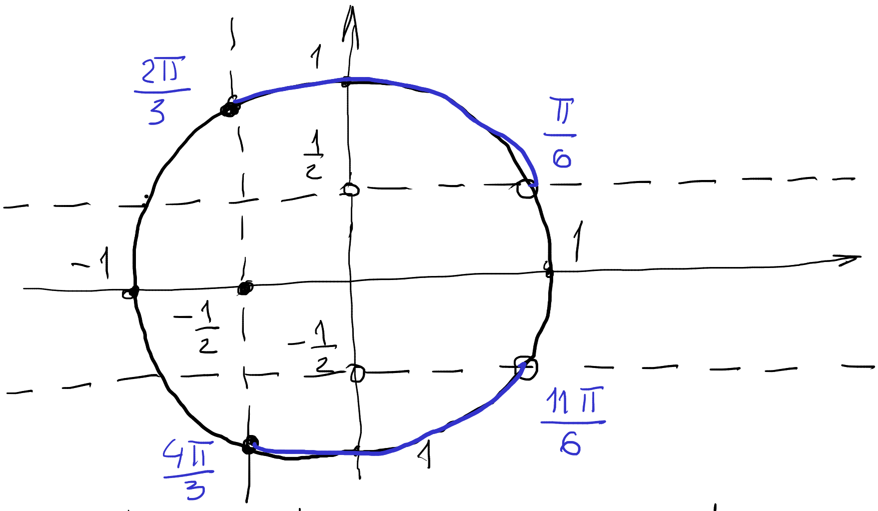
$$\boxed{a = a_I \cap a_{II} \in \left[-3, \frac{1}{3}\right]}$$

(2) (Pr 4)

$|\sin t| > \frac{1}{2}$

$\cos t \geq -\frac{1}{2}$

rješenja u intervalu $[0, 2\pi]$



I) $|\sin t| > \frac{1}{2} \Rightarrow -\sin t > \frac{1}{2}$

$\sin t < -\frac{1}{2}$ $\qquad\qquad$ $\sin t > \frac{1}{2}$

II) $\cos t \geq -\frac{1}{2}$

skup rješenja:

$$\boxed{x \in \left\langle \frac{\pi}{6}, \frac{2\pi}{3} \right] \cup \left[\frac{4\pi}{3}, \frac{11\pi}{6} \right\rangle}$$

(3) (Pr 5) $\sin t + \cos t = \frac{2}{3} \Big/ ^2$

a) $\sin^3 t + \cos^3 t = ?$

$\sin^3 t + \cos^3 t = (\sin t + \cos t) \cdot (\sin^2 t - \sin t \cos t + \cos^2 t) = (\bullet)$?

$(\sin t + \cos t)^2 = \left(\frac{2}{3}\right)^2 = \frac{4}{9}$

$\underbrace{\sin^2 t + 2\sin t\cos t + \cos^2 t}_{\sin^2 t + \cos^2 t = 1} = \frac{4}{9}$

$2\sin t\cos t = \frac{4}{9} - 1 = -\frac{5}{9} \Big/ : 2$

$\sin t \cos t = -\frac{5}{18}$

$(\bullet) = \frac{2}{3} \cdot \left(1 - \left(-\frac{5}{18}\right)\right) = \frac{\cancel{2}^{1}}{3} \cdot \frac{23}{\cancel{18}_{9}} = \boxed{\frac{23}{27}}$

b) $\sin^4 t + \cos^4 t = (\underbrace{\sin^2 t + \cos^2 t}_{=1})^2 - 2\sin^2 t\cos^2 t$

$= 1^2 - 2\left(-\frac{5}{18}\right)^2 = 1 - \cancel{2}^{1} \cdot \frac{25}{\cancel{324}_{162}} = \boxed{\frac{137}{162}}$

4 (Pr 6) $\quad tg\,t + ctg\,t = m \quad , m \neq 0$

1) $tg^3 t + ctg^3 t = ?$

$$tg^3 t + ctg^3 t = \underbrace{(tg\,t + ctg\,t)}_{=m}{}^3 \underbrace{- 3\,tg^2 t \cdot ctg\,t - 3\,tg\,t \cdot ctg^2 t}_{-3\,tg\,t\,(tg\,t\,ctg\,t - ctg^2 t)}$$

$$= m^3 - 3\overbrace{tg\,t\,ctg\,t}^{=1} \cdot \underbrace{(tg\,t + ctg\,t)}_{=m} = \boxed{m^3 - 3m}$$

2) $$\frac{1}{\sin^2 t} + \frac{1}{\cos^2 t} = ?$$

$$tg\,t + ctg\,t = m$$

$$\frac{\sin t}{\cos t} + \frac{\cos t}{\sin t} = m$$

$$\frac{\overbrace{\sin^2 t + \cos^2 t}^{=1}}{\sin t \cdot \cos t} = m$$

$$\frac{1}{\sin t \cdot \cos t} = m$$

$$\frac{1}{\sin^2 t} + \frac{1}{\cos^2 t} = \frac{\overbrace{\sin^2 t + \cos^2 t}^{=1}}{\sin^2 t \cdot \cos^2 t} = \frac{1}{\sin^2 t \cdot \cos^2 t} = \underbrace{\left(\frac{1}{\sin t \cdot \cos t}\right)}_{m}{}^2$$

$$= \boxed{m^2}$$

5 (Pr 9)

$\cos(a-b) = ?$

$\sin a + \sin b = 1 \,/^2$

$\cos a + \cos b = \sqrt{2} \,/^2$

$(\sin a + \sin b)^2 = 1$

$\sin^2 a + 2\sin a \sin b + \sin^2 b = 1 \quad (1)$

$(\cos a + \cos b)^2 = 2$

$\cos^2 a + 2\cos a \cos b + \cos^2 b = 2 \quad (2)$

$(+)$

$$\underbrace{\sin^2 a + \cos^2 a}_{=1} + \underbrace{\sin^2 b + \cos^2 b}_{=1} + 2\underbrace{(\sin a \sin b + \cos a \cos b)}_{\cos(a-b)} = 1 + 2 = 3$$

$$2+2\cos(a-b)=3$$

$$2\cos(a-b)=1$$

$$\boxed{\cos(a-b)=\frac{1}{2}}$$

6 (Pr 10)

$$\cos(\alpha+\beta)=\frac{1}{3}$$

$$\cos(\alpha-\beta)=\frac{1}{5}$$

$$tg\alpha\cdot tg\beta=?$$

$$\cos(\alpha+\beta)=\cos\alpha\cdot\cos\beta-\underline{\sin\alpha\cdot\sin\beta}=\frac{1}{3}$$

$$\cos(\alpha-\beta)=\cos\alpha\cdot\cos\beta+\underline{\sin\alpha\cdot\sin\beta}=\frac{1}{5}$$

$+$

$$2\cos\alpha\cdot\cos\beta=\frac{8}{15}\quad /:2$$

$$\cos\alpha\cos\beta=\frac{4}{15}$$

$$tg\alpha\cdot tg\beta=\frac{\sin\alpha}{\cos\alpha}\cdot\frac{\sin\beta}{\cos\beta}$$

$$\sin\alpha\cdot\sin\beta=\frac{1}{5}-\cos\alpha\cdot\cos\beta=\frac{1}{5}-\frac{4}{15}=\frac{-1}{15}$$

$$\boxed{tg\alpha\cdot tg\beta=}\ \frac{\frac{-1}{15}}{\frac{4}{15}}=\boxed{-\frac{1}{4}}$$

7 (Pr 13)

$$f\left(x+\frac{3\pi}{2}\right)=\sin x+\cos x$$

$$f\left(\frac{\pi}{2}-x\right)\cdot f\left(\frac{\pi}{2}+x\right)=\cos 2x$$

Dokazati: $f\left(x+\frac{3\pi}{2}\right)=\sin x+\cos x=\cos\left(x+\frac{3\pi}{2}\right)-\sin\left(x+\frac{3\pi}{2}\right)$

$$f(x)=\cos x-\sin x$$

$$f\left(\frac{\pi}{2}-x\right)\cdot f\left(\frac{\pi}{2}+x\right)=\left[\cos\left(\frac{\pi}{2}-x\right)-\sin\left(\frac{\pi}{2}-x\right)\right]\cdot$$

$$\cdot\left[\cos\left(\frac{\pi}{2}+x\right)+\sin\left(\frac{\pi}{2}+x\right)\right]=$$

$= (\sin x - \cos x) \cdot (-\sin x - \cos x) = -\sin^2 x \underline{- \cancel{\sin x \cos x} +}$
$+ \cancel{\sin x \cdot \cos x} + \cos^2 x = \boxed{\cos(2x)}$

8 (Pr 16/2)

$$\sin\left(\frac{17\pi}{24}\right) \cdot \sin\left(\frac{23\pi}{24}\right)$$

$$\underline{\sin x \cdot \sin y = \frac{1}{2}\left[\cos(x-y) - \cos(x+y)\right]}$$

$$\boxed{\sin\left(\frac{17\pi}{24}\right) \cdot \sin\left(\frac{23\pi}{24}\right)} = \frac{1}{2}\left[\cos\left(\frac{17\pi - 23\pi}{24}\right) - \cos\left(\frac{17\pi + 23\pi}{24}\right)\right]$$

$$= \frac{1}{2}\left[\cos\left(-\frac{\overset{1}{\cancel{6}}\pi}{\underset{4}{\cancel{24}}}\right) - \cos\left(\frac{\overset{5}{\cancel{40}}\pi}{\underset{3}{\cancel{24}}}\right)\right] = \frac{1}{2}\left[\frac{\sqrt{2}}{2} - \frac{1}{2}\right] = \boxed{\frac{\sqrt{2}-1}{4}}$$

9 (Pr 15)

$$2\,ctg^2 x + 7\,ctg\, x + 3 = 0$$

$$\frac{3\pi}{2} < x < \frac{7\pi}{4}$$

$$\cos 2x = ?$$

supstitucija $t = ctg\, x$

$$2t^2 + 7t + 3 = 0 \quad \begin{matrix} \nearrow t_1 = -3 \\ \searrow t_2 = -\frac{1}{2} \end{matrix}$$

I) $ctg\, x = -3$ ⊖

II) $ctg\, x = -\frac{1}{2}$ ✓

$$\cos(2x) = \cos^2 x - \sin^2 x$$

$$\underbrace{\downarrow \qquad \downarrow}_{= f(ctg\, x) = ?}$$

$$\cos x = \frac{ctg\, x}{\sqrt{1 + ctg^2 x}}$$

$$\cos x = \frac{-\frac{1}{2}}{\sqrt{1+\left(-\frac{1}{2}\right)^2}} = \frac{-\frac{1}{2}}{\sqrt{\frac{5}{4}}}$$

$$= -\frac{\sqrt{5}}{5}$$

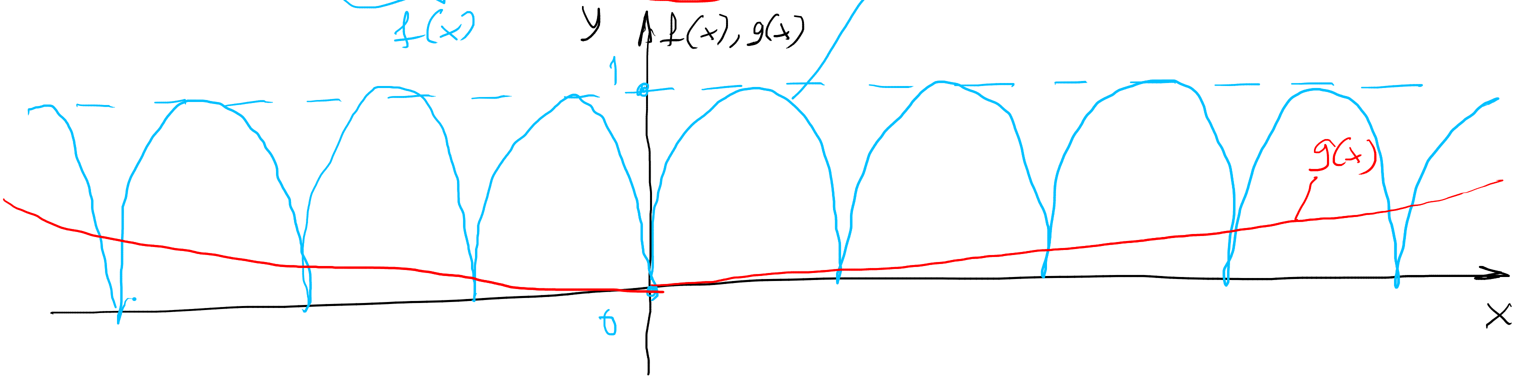
$$\sin x = -\sqrt{1-\cos^2 x} = -\sqrt{1-\left(-\frac{\sqrt{5}}{5}\right)^2} = -\frac{2}{\sqrt{5}} = -\frac{2\sqrt{5}}{5}$$

$$\boxed{\cos 2x} = \left(-\frac{\sqrt{5}}{5}\right)^2 - \left(-\frac{2\sqrt{5}}{5}\right)^2 = \frac{5}{25} - \frac{20}{25} = -\frac{15}{25} = -\frac{3}{5}$$

11. TRIGONOMETRIJSKE JEDNADŽBE I NEJEDNADŽBE

① (Pr 6)

$$\underbrace{|\sin 2x|}_{f(x)} = \underbrace{\frac{1}{100}x^2}_{g(x)}$$



$g(x) > 1$

$\frac{1}{100}x^2 > 1$

$x^2 > 100$

$x > 10 \quad / : \frac{\pi}{2}$

$\frac{x}{\frac{\pi}{2}} = \frac{10}{\frac{\pi}{2}} \approx 6{,}37$

↳ gledamo samo cijeli broj, broj punih puta $= 6$

$n = 6 \cdot \underbrace{2}_{} \cdot 2 - 1 = 24 - 1 = 23$

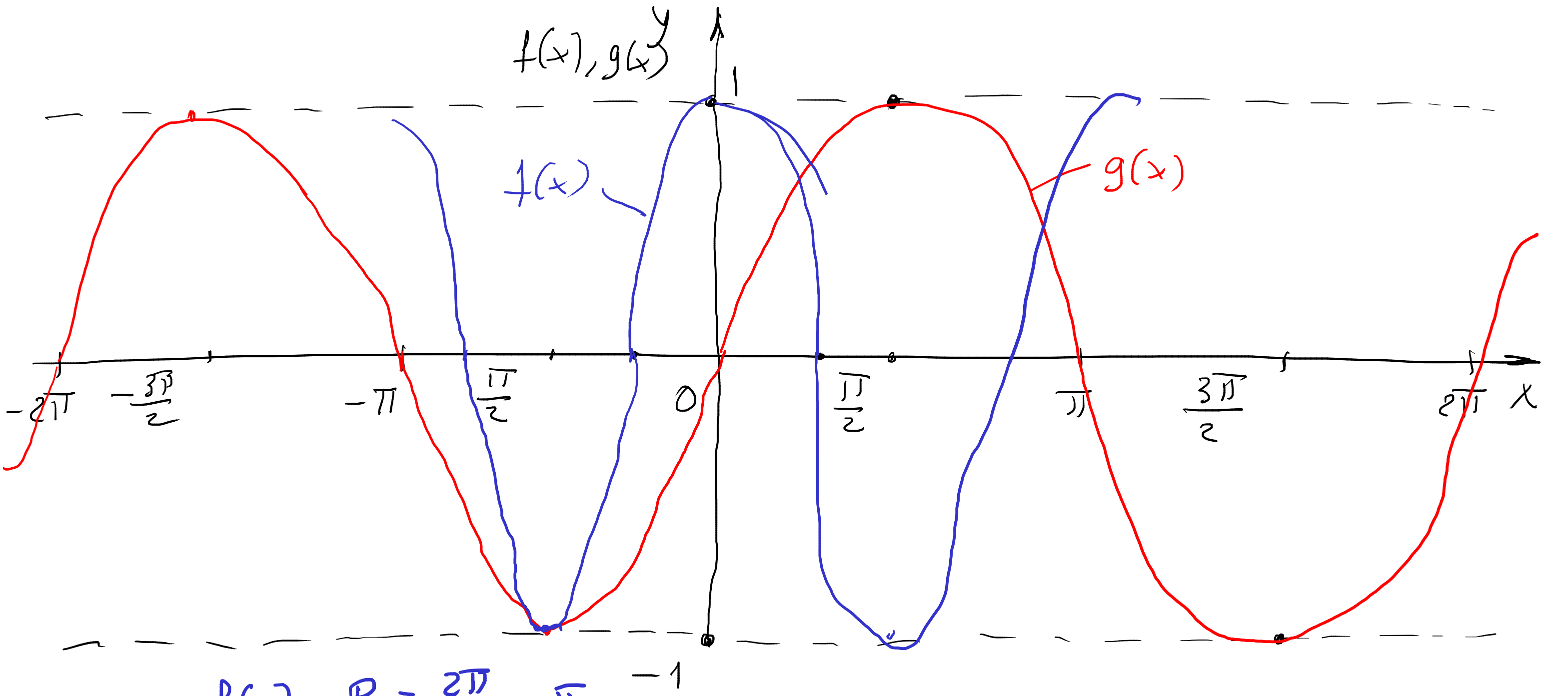
→ lijevo i desno od nule ($x < 0$, $x > 0$)

→ ishodište ne računam, tjeme parabole!

↳ $2x$ presijeca poluperiod sinusoide

② (Pr 7)

$$\underbrace{\cos 2x}_{f(x)} \le \underbrace{\sin x}_{g(x)} \qquad x \in [0, 2\pi]$$



za $f(x)$ $\quad P = \frac{2\pi}{2} = \pi$

$\cos 2x \leq \sin x$

$\cos 2x - \sin x \leq 0$

$2\sin^2 x + \sin x - 1 = 0$ $\qquad\qquad\qquad\qquad f(x) > g(x)$

$\sin x = t$

$2t^2 + t - 1 = 0 \quad \begin{cases} t_1 = -1 \\ t_2 = \frac{1}{2} \end{cases}$

$\sin x = -1 \Rightarrow x_1 = \frac{3\pi}{2}$ $\ominus$

$\sin x = \frac{1}{2} \Rightarrow \boxed{\begin{matrix} x_2 = \frac{\pi}{6} \\ x_3 = \frac{5\pi}{6} \end{matrix}}$ $\oplus$

③ (Pr 8)

$\sqrt{3} \cdot \operatorname{ctg}\left(x - \frac{\pi}{6}\right) = 1 \quad / : \sqrt{3}$

$\operatorname{ctg}\left(x - \frac{\pi}{6}\right) = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$

$x - \frac{\pi}{6} = \frac{\pi}{3} + k \cdot \pi$

$\boxed{x} = \frac{\pi}{3} + \frac{\pi}{6} + k \cdot \pi = \frac{\cancel{3}^{1}\pi}{\cancel{6}_{2}} + k \cdot \pi = \boxed{\frac{\pi}{2} + k \cdot \pi, \quad k \in \mathbb{Z}}$

④ (Pr 10)

$$2\cos^2 x + 3\sin x \cdot \cos x + 3\sin^2 x = \underbrace{1}_{\sin^2 x + \cos^2 x}$$

$$2\cos^2 x - \cos^2 x + 3\sin x \cdot \cos x + 3\sin^2 x - \sin^2 x = 0$$

$$\cos^2 x + 2\sin^2 x + 3\sin x \cdot \cos x = 0 \quad / : \sin^2 x$$

$$\underbrace{\frac{\cos^2 x}{\sin^2 x}}_{ctg^2 x} + 2 + 3\overbrace{\frac{\cancel{\sin x}^{\,1}}{\sin^2 x}\cos x}^{ctg\, x} = 0$$

$$ctg^2 x + 3ctg\, x + 2 = 0$$

supstitucija $t = ctg\, x$

$$t^2 + 3t + 2 = 0 \quad \begin{matrix} \nearrow t_1 = -2 \\ \searrow t_2 = -1 \end{matrix}$$

I) za $t_1 = -2$ $\quad ctg\, x = -2$

$$\boxed{x_1 \approx -0{,}46 + k \cdot \pi, \ k \in \mathbb{Z}}$$

II) za $t_2 = -1$ $\quad ctg\, x = -1$

$$\boxed{x_2 = \frac{3\pi}{4} + k \cdot \pi, \ k \in \mathbb{Z}}$$

⑤ (Pr 13)

$$3\sin x + 2\cos x = 3 \quad (*)$$

$$\sin x = \frac{2\,tg\left(\frac{x}{2}\right)}{1 + tg^2\left(\frac{x}{2}\right)} \quad \rightarrow \text{trig. funkcije dvostrukog argumenta}$$

$$\cos x = \frac{1 - tg^2\left(\frac{x}{2}\right)}{1 + tg^2\left(\frac{x}{2}\right)}$$

supstitucija $t = tg\left(\frac{x}{2}\right)$

$$\sin x = \frac{2t}{1+t^2} \qquad \cos x = \frac{1-t^2}{1+t^2} \longrightarrow (*)$$

$$3\cdot\frac{2t}{1+t^2}+2\,\frac{1-t^2}{1+t^2}=3 \quad /\cdot(1+t^2)$$

$$3\cdot 2t+2(1-t^2)=3(1+t^2)$$

$$6t+2-2t^2=3+3t^2$$

$$-5t^2+6t-1=0 \quad /\cdot(-1)$$

$$5t^2-6t+1=0$$

$$t_1=\frac{1}{5}$$

$$t_2=1$$

I) $t_1=\frac{1}{5}$ $\qquad tg\left(\frac{x_1}{2}\right)=\frac{1}{5}$

$$x_1=2\left(arctg\left(\tfrac{1}{5}\right)+k\cdot\pi\right)$$

$$\boxed{x_1=2\,arctg(0.2)+2\cdot k\cdot\pi,\quad k\in\mathbb{Z}}$$

II) $t_2=1$ $\qquad tg\left(\frac{x_2}{2}\right)=1$

$$\boxed{x_2=\frac{\pi}{2}+k\cdot 2\pi,\quad k\in\mathbb{Z}}$$

6 (Pr 18) $\qquad \sin x+\sqrt{3}\cos x>0 \qquad \langle 0, 2\pi\rangle$

$\sin x>-\sqrt{3}\cos x \quad (1)$

I) $\cos x>0$

$(1)\ /:\cos x$

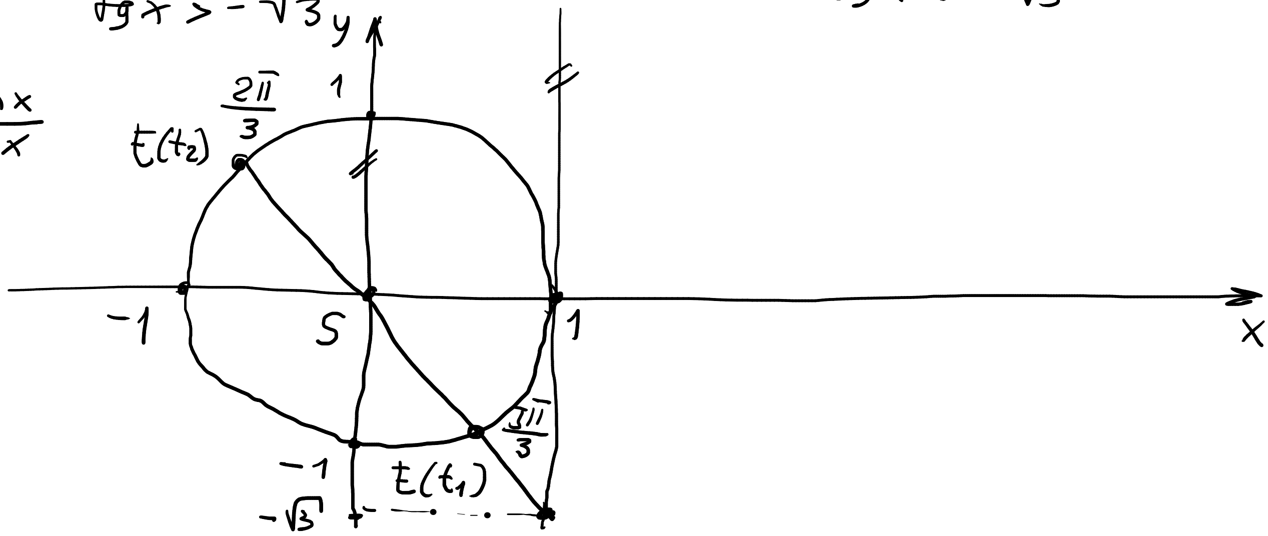
$tg\,x>-\sqrt{3}$

II) $\cos x<0$

$(1)\ /:\cos x$

$tg\,x<-\sqrt{3}$

$tg\,x=\frac{\sin x}{\cos x}$



I) $x_I = \langle 0, \frac{\pi}{2} \rangle \cup \langle \frac{5\pi}{3}, 2\pi \rangle$

II) $x_{II} \in \langle \frac{\pi}{2}, \frac{2\pi}{3} \rangle$

$\boxed{x \in \langle 0, \frac{2\pi}{3} \rangle \cup \langle \frac{5\pi}{3}, 2\pi \rangle}$

7) $2\cos^2 x + \cos x \geq 1$

$2\cos^2 x + \cos x - 1 \geq 0$

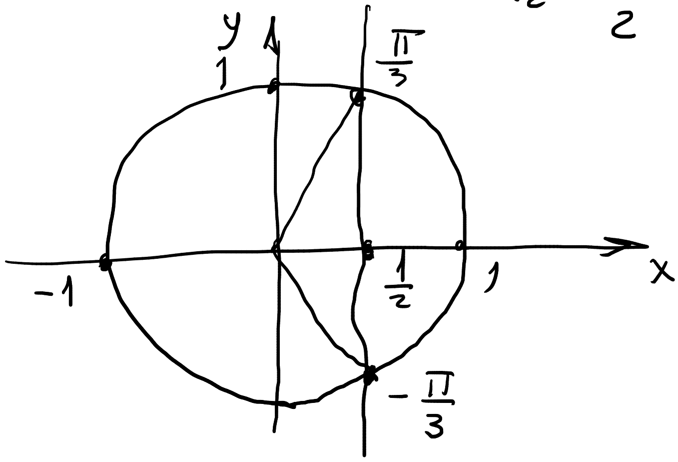
$t = \cos x$

$2t^2 + t - 1 = 0 \rightarrow t_1 = -1, \quad t_2 = \frac{1}{2}$

$f(t) \geq 0$

$\cos x \geq -1$

$\cos x \geq \frac{1}{2}$



$\boxed{-\frac{\pi}{3} + k \cdot 2\pi \leq x \leq \frac{\pi}{3} + k \cdot 2\pi}$

12. PLANIMETRIJA

1) (Pr 2)

$\alpha : \beta = 1 : 2$

$\beta : \gamma = 4 : 9$

$\alpha, \beta, \gamma = ?$

trokut

$\frac{\alpha}{\beta} = \frac{1}{2} \Rightarrow \beta = 2\alpha$

$\frac{\beta}{\gamma} = \frac{4}{9} \Rightarrow \gamma = \frac{9}{4}\beta = \frac{9}{4} \cdot 2\alpha = \frac{9}{2}\alpha$

$\alpha + \beta + \gamma = 180^\circ$

$\alpha + 2\alpha + \frac{9}{2}\alpha = 180^\circ$

$\frac{2 + 4 + 9}{2}\alpha = 180^\circ$

$$\frac{15}{2}\alpha = 180^\circ \quad / \cdot \frac{2}{15}$$

$$\boxed{\alpha = \frac{180^\circ \cdot 2}{15} = \frac{360^\circ}{15} = 24^\circ}$$

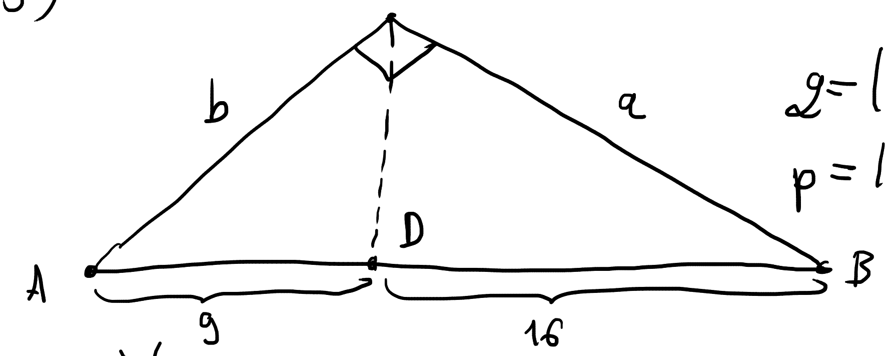
$$\boxed{\beta = 2\alpha = 2 \cdot 24 = 48^\circ}$$

$$\boxed{\gamma = \frac{9}{2} \cdot 24 = 108^\circ}$$

provjera:

$24^\circ + 48^\circ + 108^\circ = 180^\circ$ ✓

② (Pr 5)



$a, b = ?$

$q = |\overline{AD}| = 9\ cm$

$p = |\overline{BD}| = 16\ cm$

$c = 9 + 16 = 25\ cm$

Euklidov poučak

$$\boxed{a = \sqrt{c \cdot p} = \sqrt{25 \cdot 16} = 20\ cm}$$

$$\boxed{b = \sqrt{c \cdot q} = \sqrt{25 \cdot 9} = 15\ cm}$$

$$\boxed{a + b = 20 + 15 = 35\ cm}$$

③ (Pr 8)

trapez

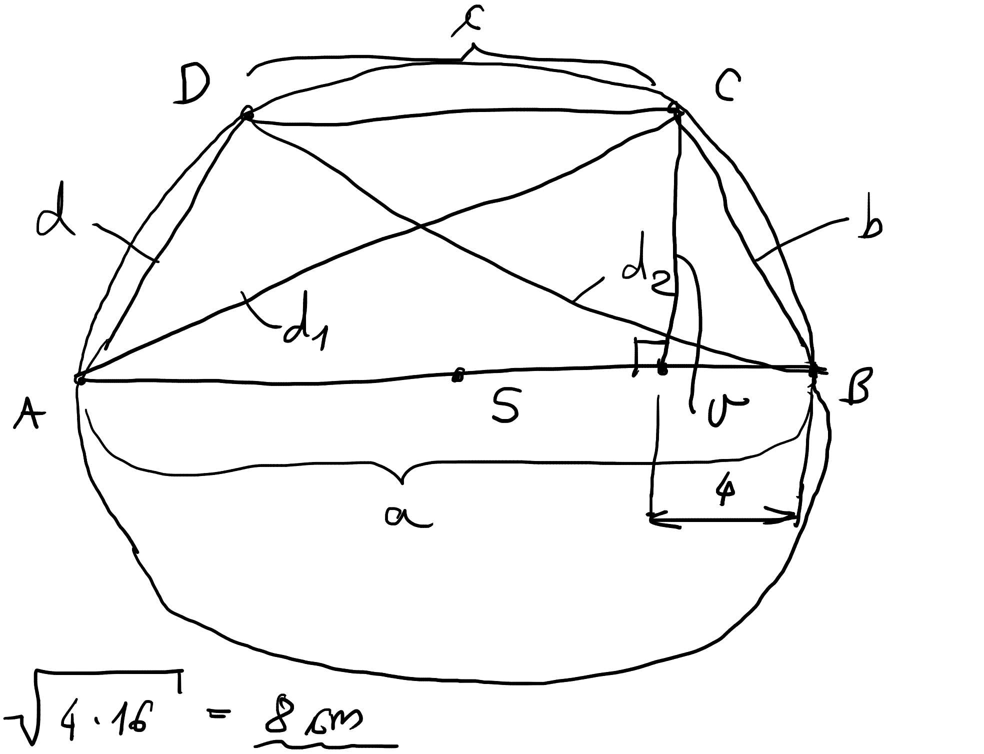
$a = 20\ cm = |\overline{AB}|$

$c = 12\ cm = |\overline{CD}|$

$P = ?$

$b, d = ?$

$d_1, d_2 = ?$



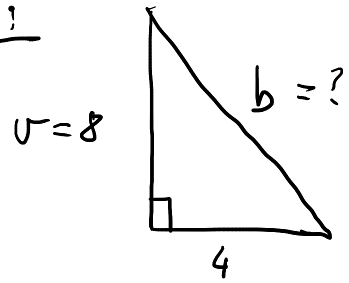
a) Površina trapeza

$$P = \frac{a + c}{2} \cdot v$$

Euklidov poučak: $v = \sqrt{4 \cdot 16} = \underline{8\ cm}$

$$\boxed{P = \frac{20 + 12}{2} \cdot 8 = 128\ cm^2}$$

b) Duljine krakova:

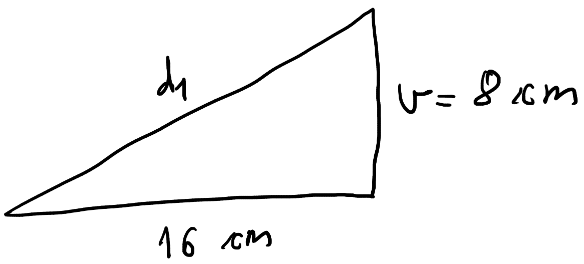


$$b = \sqrt{8^2 + 4^2}$$

$$\boxed{b = \sqrt{80} = 4\sqrt{5}\ cm}$$

$$\boxed{d = b = 4\sqrt{5}\ cm}$$

c) Dijagonala trapeza d_1:



$$d_1 = \sqrt{8^2 + 16^2}$$

$$d_1 = \sqrt{64 + 256} = \sqrt{320}$$

$$\boxed{d_1 = 8\sqrt{5}\ cm = d_2}$$

4) ~~sličnost trokuta~~

trokut 1: 52°, 80°

trokut 2: 48°, 80°

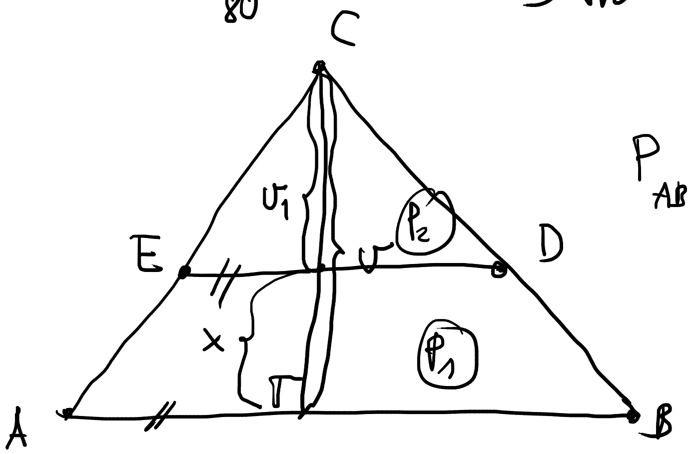
$$180^\circ - (52^\circ + 80^\circ) = 180^\circ - 132^\circ = \underline{48^\circ}$$

$$180^\circ - (48^\circ + 80^\circ) = 52^\circ$$

48° 52° 80° svi kutevi su jednaki $\Rightarrow$ trokuti su slični!

5) ΔABC

$v = 10\ cm$



$$P_{ABC} = P_1 + P_2$$

$$x = v - v_1$$

$$\Delta ABC \sim \Delta DEC$$

koeficijent sličnosti k:

$$k^2 = \frac{P}{P_1} = 2 \Big/ ^{\frac{1}{2}} \Rightarrow \underline{k = \sqrt{2}}$$

$$k = \frac{v}{v_1} = \frac{10}{v_1} = \sqrt{2}$$

$$\Rightarrow \underline{v_1} = \frac{10}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{10\sqrt{2}}{2} = \underline{5\sqrt{2}\ cm}$$

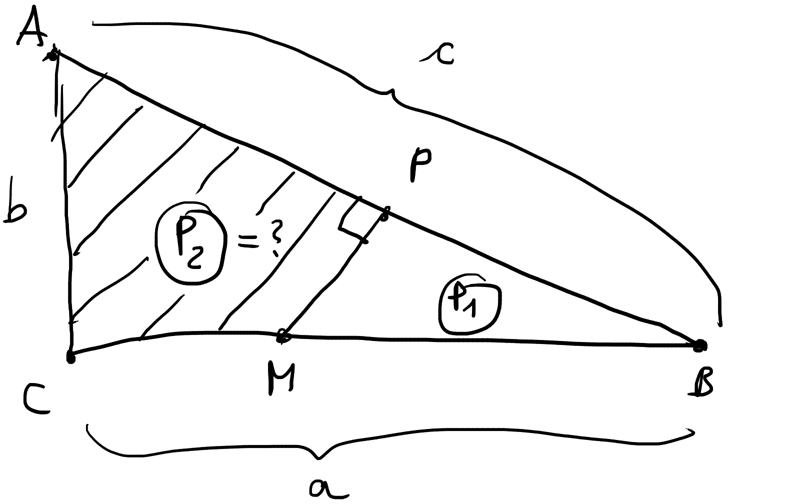
$$\boxed{x = v - v_1 = 10 - 5\sqrt{2} = 5(2-\sqrt{2})\ \text{cm}}$$

6 (Pr 14)

pravokutni trokut

$a = 5$ cm

$b = 3$ cm



$\Delta ABC \sim \Delta BMP$

$$\boxed{P = \frac{a \cdot b}{2} = \frac{5 \cdot 3}{2} = \frac{15}{2}\ \text{cm}^2}$$

$$c = \sqrt{5^2 + 3^2} = \sqrt{25+9} = \sqrt{34}\ \text{cm}$$

$$P_1 = \frac{|\overline{BM}|^{?} \cdot |\overline{BP}|}{2}$$

$$|\overline{BP}| = \frac{c}{2} = \frac{\sqrt{34}}{2}\ \text{cm}$$

preko sličnosti trokuta:

$$\frac{\overline{BP}}{\overline{MP}} = \frac{a}{b}$$

$$\Rightarrow \overline{MP} = \frac{b}{a} \cdot \overline{BP} = \frac{3}{5} \cdot \frac{\sqrt{34}}{2} = \frac{3\sqrt{34}}{10}\ \text{cm}$$

$$P_1 = \frac{1}{2} \cdot \frac{3\sqrt{34}}{10} \cdot \frac{\sqrt{34}}{2} = \frac{3 \cdot 34}{4 \cdot 10} = \frac{102}{40} = \frac{51}{20}\ \text{cm}^2$$

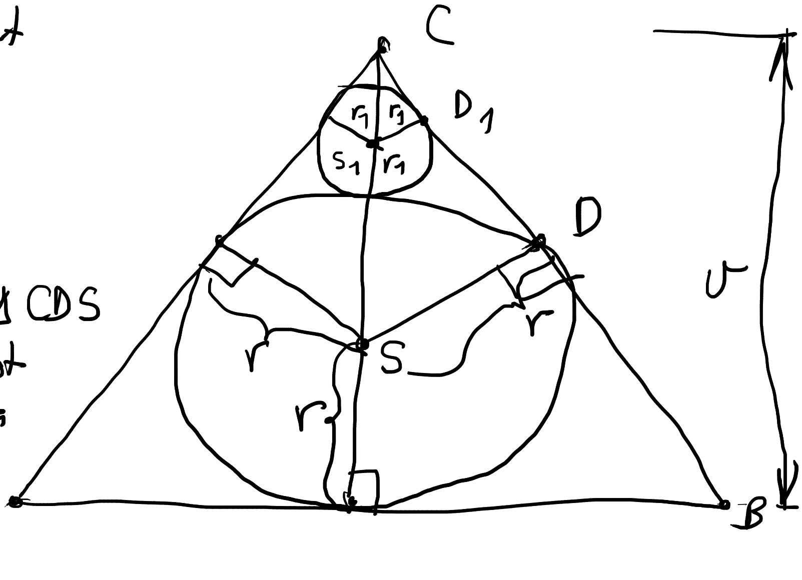
$$\boxed{P_2 = P - P_1 = \frac{15}{2} - \frac{51}{20} = \frac{150-51}{20} = \frac{99}{20}\ \text{cm}^2 \approx 4{,}95\ \text{cm}^2}$$

7 (Pr 15) jednakokračni trokut

$v = 8$ cm

$r = 2$ cm

$r_1 = ?$



$$\frac{\overline{CS_1}}{r_1} = \frac{\overline{CS}}{r}$$ ← $\Delta CS_1D_1 \sim \Delta CDS$ sličnost trokuta

$$|\overline{CS}| = v - r = 8 - 2 = 6\ \text{cm}$$

$$\left|\overline{CS_1}\right| = -(r_1 + 2r) + v = 8 - (r_1 + 2 \cdot 2) = 8 - r_1 - 4 = 4 - r_1$$

$$\frac{\overline{CS_1}}{r_1} = \frac{6}{2} = 3 = \frac{\overline{CS}}{r} \Rightarrow \left|\overline{CS_1}\right| = 3 r_1$$

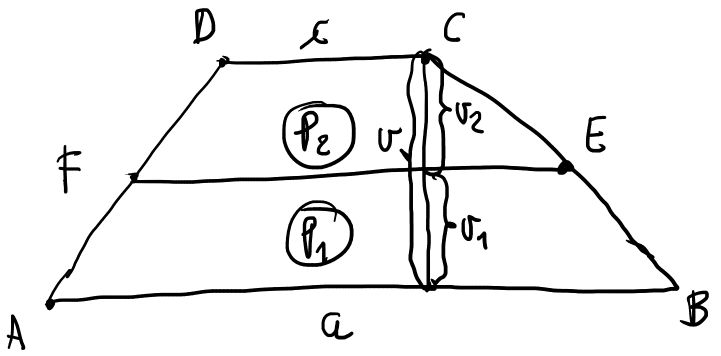
$(\Leftrightarrow)$

$$4 - r_1 = 3 r_1$$

$$-4 r_1 = -4 \Rightarrow \boxed{r_1 = 1 \text{ cm}}$$

(8) trapez

$c = 1$ cm
$a = 3$ cm



$$P_{ABCD} = P_1 + P_2$$

$$P_{ABCD} = \frac{a+c}{2} \cdot v$$

$$= \frac{3+1}{2} \cdot (v_1 + v_2) = 2(v_1 + v_2)$$

$$P_{ABCD} = 2 \cdot P_2 = 2\,\frac{x+c}{2} \cdot v_2 = \cancel{2}\,\frac{x+1}{\cancel{2}} \cdot v_2 = (x+1)\,v_2$$

$(\Leftrightarrow)$

$$|\overline{EF}| = x$$

$$2(v_1 + v_2) = (x+1)\,v_2 \quad / : v_2$$

$$P_1 = P_2$$

$$P_1 = \frac{3+x}{2} \cdot v_1$$
$$P_2 = \frac{x+1}{2}\,v_2$$

$(:)$

$$\frac{3+x}{x+1} = \frac{v_1}{v_2}$$

$$\Rightarrow x + 1 = (3+x) \cdot \frac{v_1}{v_2}$$

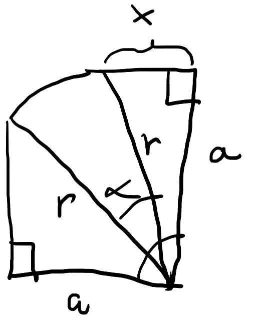
$$2\left(\frac{v_1}{v_2} + 1\right) = x + 1$$

$$2\left(\frac{v_1}{v_2} + 1\right) = (3+x)\,\frac{v_1}{v_2} \quad \dots \quad \boxed{x = \sqrt{5} \text{ cm}}$$

9 (Pr 19)

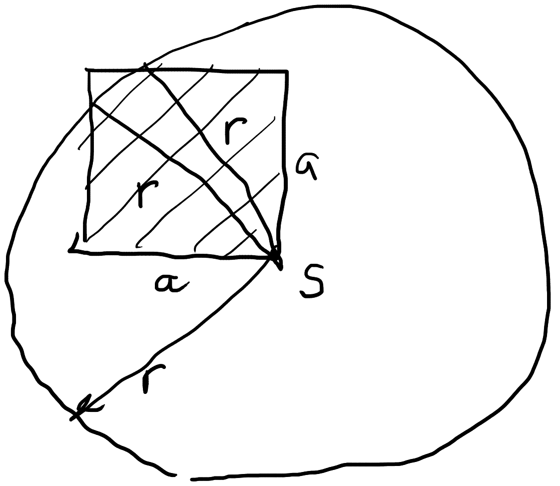
$a = 3\,cm$

$r = 2\sqrt{3}\,cm$



$90^\circ = 3\alpha$

$\Rightarrow \alpha = 30^\circ$



$x = \sqrt{(2\sqrt{3})^2 - 3^2} = \sqrt{3}\,cm$

$P_{uk} = P_{kl} + 2P_\Delta$

$P_\Delta = \frac{a \cdot x}{2} = \frac{3 \cdot \sqrt{3}}{2}\,cm$

$P_{kl} = \frac{r^2 \pi \alpha}{360^\circ} = \frac{(2\sqrt{3})^2 \pi \cdot 30^\circ}{360^\circ} = \frac{360^\circ \cdot \pi}{360^\circ} = \pi\,cm^2$

$$\boxed{P_{uk} = \pi + 2 \cdot \frac{3\sqrt{3}}{2} = \pi + 3\sqrt{3}\,cm^2}$$

10 (zadni - 31)

trapez ABCD

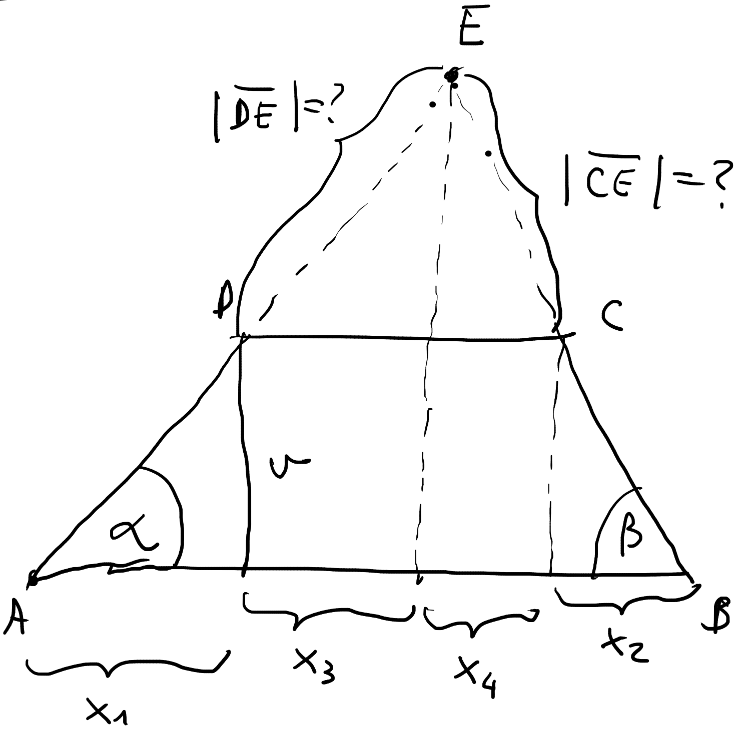
$|AB| = 12\,cm$

$|BC| = 5\,cm$

$|CD| = 8\,cm$

$|AD| = 6\,cm$

$|DE| = ?,\ |CE| = ?$



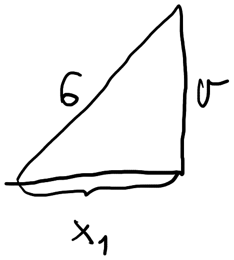
prema Euklidovom poučku:

$v = \sqrt{(12-x_1)\,x_1} = \sqrt{(12-x_2)\,x_2} \quad /^2$

$v^2 = 12x_1 - x_1^2 = 12x_2 - x_2^2 \quad (1)$

$a = c + x_1 + x_2$

$12 = 8 + x_1 + x_2 \implies x_1 + x_2 = 12 - 8 = 4 \quad (2)$



$v = \sqrt{6^2 - x_1^2} = \sqrt{36 - x_1^2} \quad (3) \quad |^2$

$v^2 = 36 - x_1^2$

$12x_1 - \cancel{x_1^2} = 36 - \cancel{x_1^2}$

$x_1 = 3\text{ cm}$

$x_2 = 12 - 8 - 3 = 1\text{ cm}$

$x_3 + x_4 = 8 \longrightarrow 4x_4 = 8$

$\frac{x_3}{x_4} = \frac{3}{1} \implies x_3 = 3x_4$

$x_4 = 2\text{ cm}$

$x_3 = 6\text{ cm}$

$\sin\alpha = \frac{x_1}{|AD|} = \frac{3}{6} = \frac{1}{2}$

$\sin\beta = \frac{x_2}{|BC|} = \frac{1}{5}$

$\sin\alpha = \frac{x_3}{|DE|} \implies \boxed{|DE|} = \frac{x_3}{\sin\alpha} = \frac{6}{\frac{1}{2}} = \boxed{12\text{ cm}}$

$\boxed{|CE|} = \frac{x_4}{\underbrace{\sin\beta}_{=\frac{1}{5}}} = \frac{2}{\frac{1}{5}} = \boxed{10\text{ cm}}$