19. ELIPSA, HIPERBOLA I PARABOLA

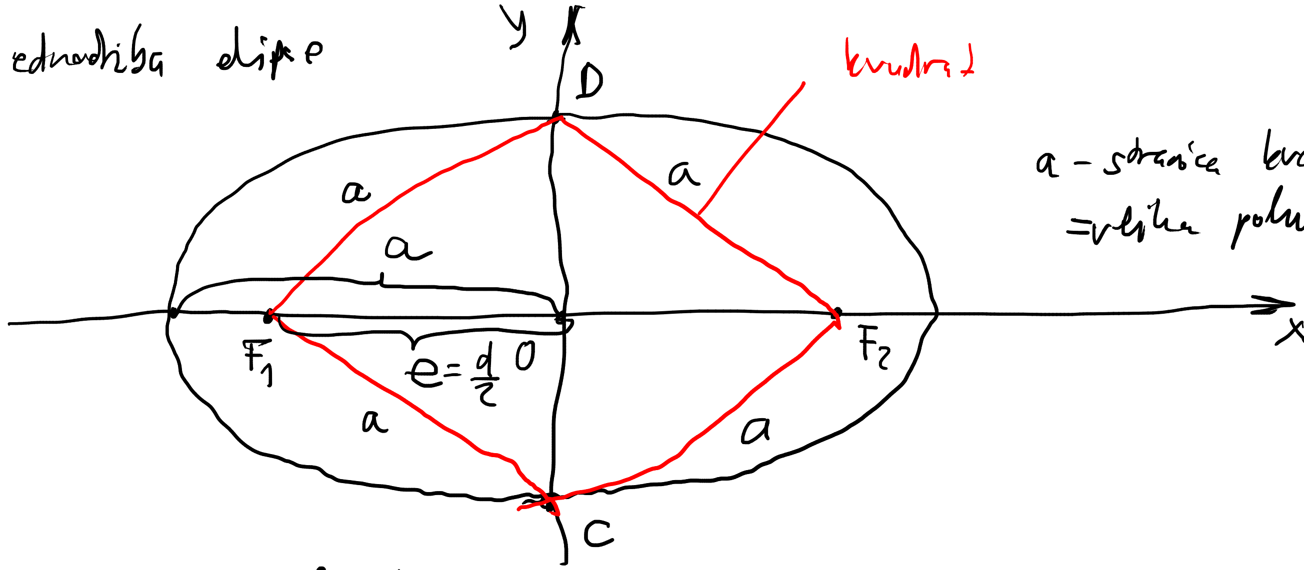
① (Pr 3)

elipsa

$F_1(-4,0)$, $F_2(4,0)$ } žarišta elipse } vrhovi kvadrata

točke C, D – mala os – vrhovi kvadrata

jednadžba elipse



a – stranica kvadrata = velika poluos

$\overline{F_1F_2}$ – dijagonala kvadrata

$|F_1F_2| = d = 2e = 8 \Rightarrow e = 4$

kvadrat F_1CF_2D : $a^2+a^2=d^2$

$2a^2 = d^2$

$\Rightarrow a^2 = \frac{d^2}{2} = \frac{8^2}{2} = 32 \quad / \sqrt{\ }$

$a = \sqrt{32} = \sqrt{2\cdot 16} = 4\sqrt{2}$

jednadžba elipse: $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ ✓ ?

$|CD| = 2b = d = 8 \Rightarrow b = 4$

$b^2 = 16$

$$\boxed{\frac{x^2}{32} + \frac{y^2}{16} = 1}$$

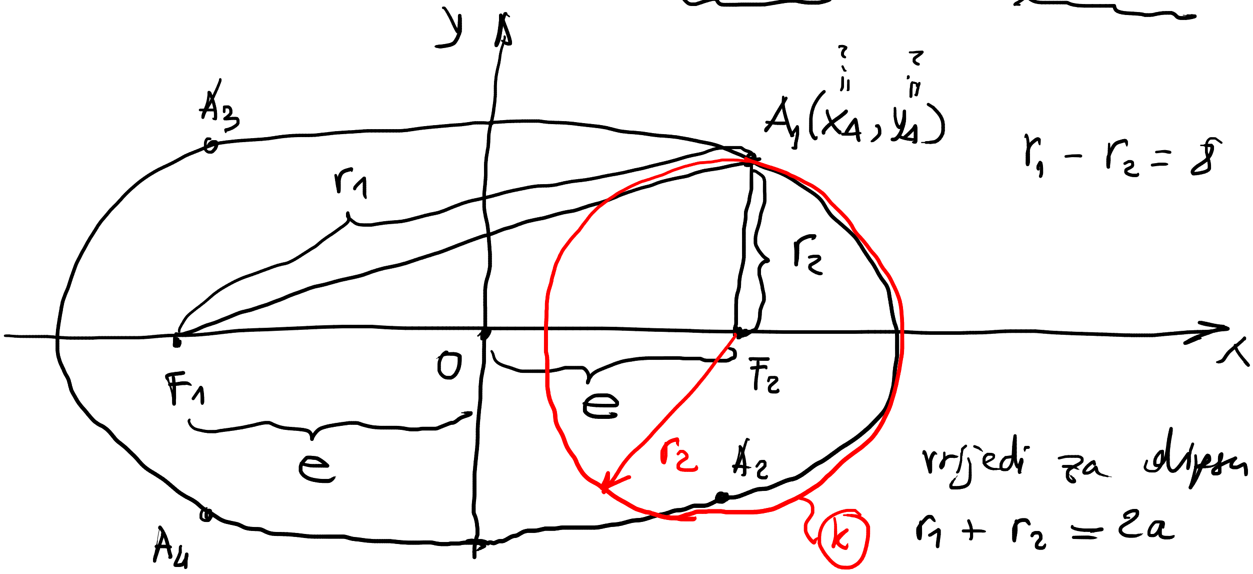
② (Pr 4)

e ... $5x^2 + 9y^2 = 405 \;/ : 405$

$$\frac{\cancel{5}^{1}x^2}{\cancel{405}_{81}} + \frac{\cancel{9}^{1}y^2}{\cancel{405}_{45}} = 1 \qquad \underbrace{\frac{x^2}{81} + \frac{y^2}{45} = 1}$$

$\Rightarrow a^2 = 81 \;/\sqrt{\;}$ $\qquad b^2 = 45 = 5 \cdot 9 \;/\sqrt{\;}$

$\underline{a = 9}$ $\qquad \underline{b = 3\sqrt{5}}$



$r_1 - r_2 = 8 \quad (1)$

vrijedi za elipsu:

$r_1 + r_2 = 2a$

$r_1 + r_2 = 2 \cdot 9 = 18 \quad (2)$

(+)

$2r_1 = 26 \Rightarrow \underline{r_1 = 13}$

$\underline{r_2 = 5}$

$F_1(-e, 0)$?

$F_2(e, 0)$

$a^2 - b^2 = e^2$

$e^2 = 81 - 45 = 36 \;/\sqrt{\;} \Rightarrow e = 6$

$F_1(-6, 0)$

$F_2(6, 0)$

1) k ... $(x-6)^2 + y^2 = 25$

$x^2 - 12x + 36 + y^2 = 25$

$x^2 - 12x + 11 + y^2 = 0$

A

e ... $5x^2 + 9y^2 = 405 \Rightarrow y^2 = 45 - \frac{5}{9}x^2$

$\frac{4}{9}x^2 - 12x + 56 = 0$

$(x_A)_1 = 6 \Rightarrow (y_A)_1 = -5$

$(x_A)_2 = -6 \Rightarrow (y_A)_2 = 5$

Rješenja:

$A_1(6,5)$	$A_3(-6,5)$
$A_2(6,-5)$	$A_4(-6,-5)$

3 h ... $x^2-y^2-4x-6y-9=0$

nadopuna na potpuni kvadrat

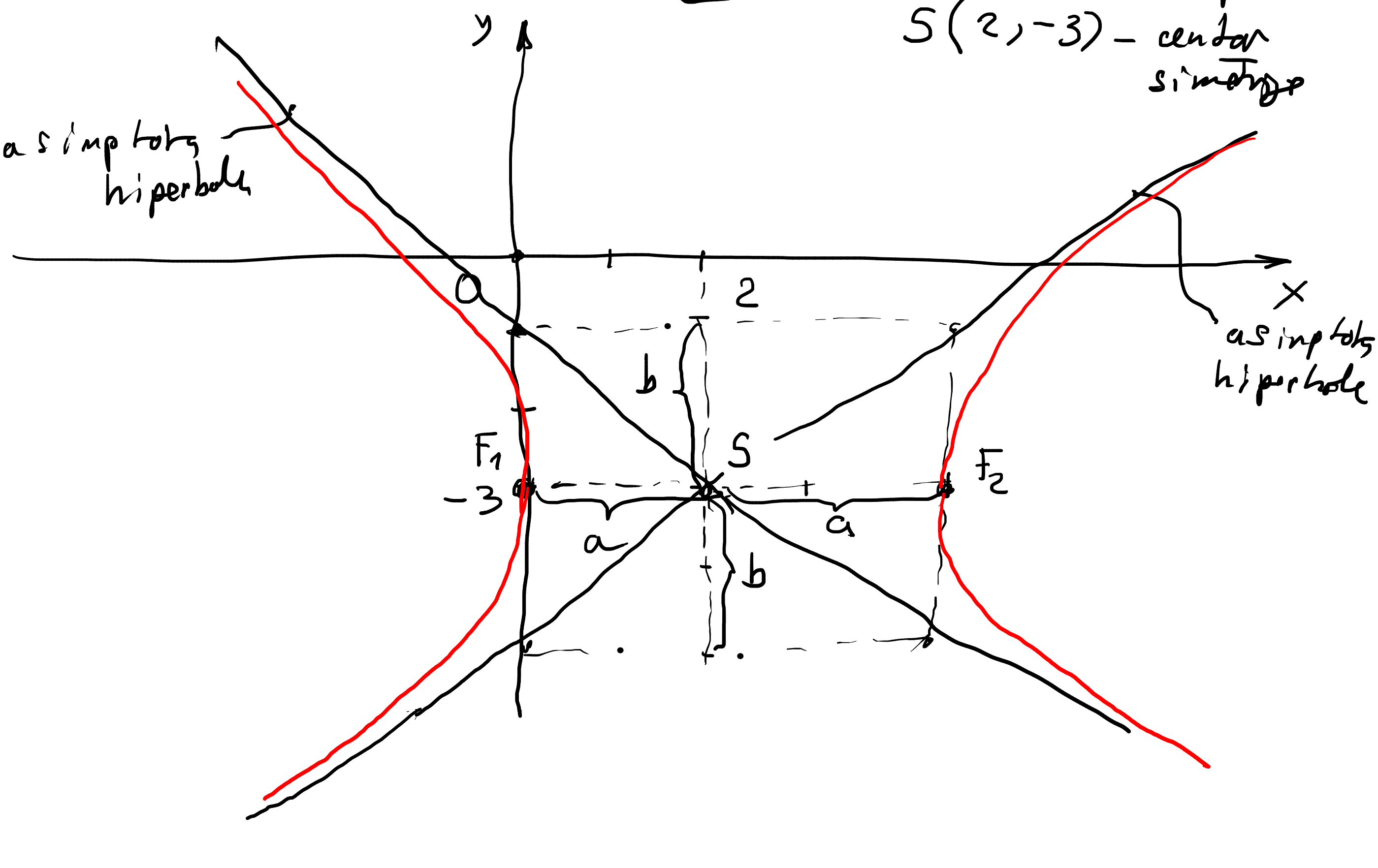
$$x^2-4x-(y^2+6y)=9$$

$$(x-2)^2-(y+3)^2=9+4-9=4 \quad /:4$$

$$\frac{(x-2)^2}{4}-\frac{(y+3)^2}{4}=1 \implies a=b=2$$

jednakostranična hiperbola

$S(2,-3)$ – centar simetrije



4 (pr 7)

(h) ... $9x^2 - 16y^2 = 144 \quad / : 144$

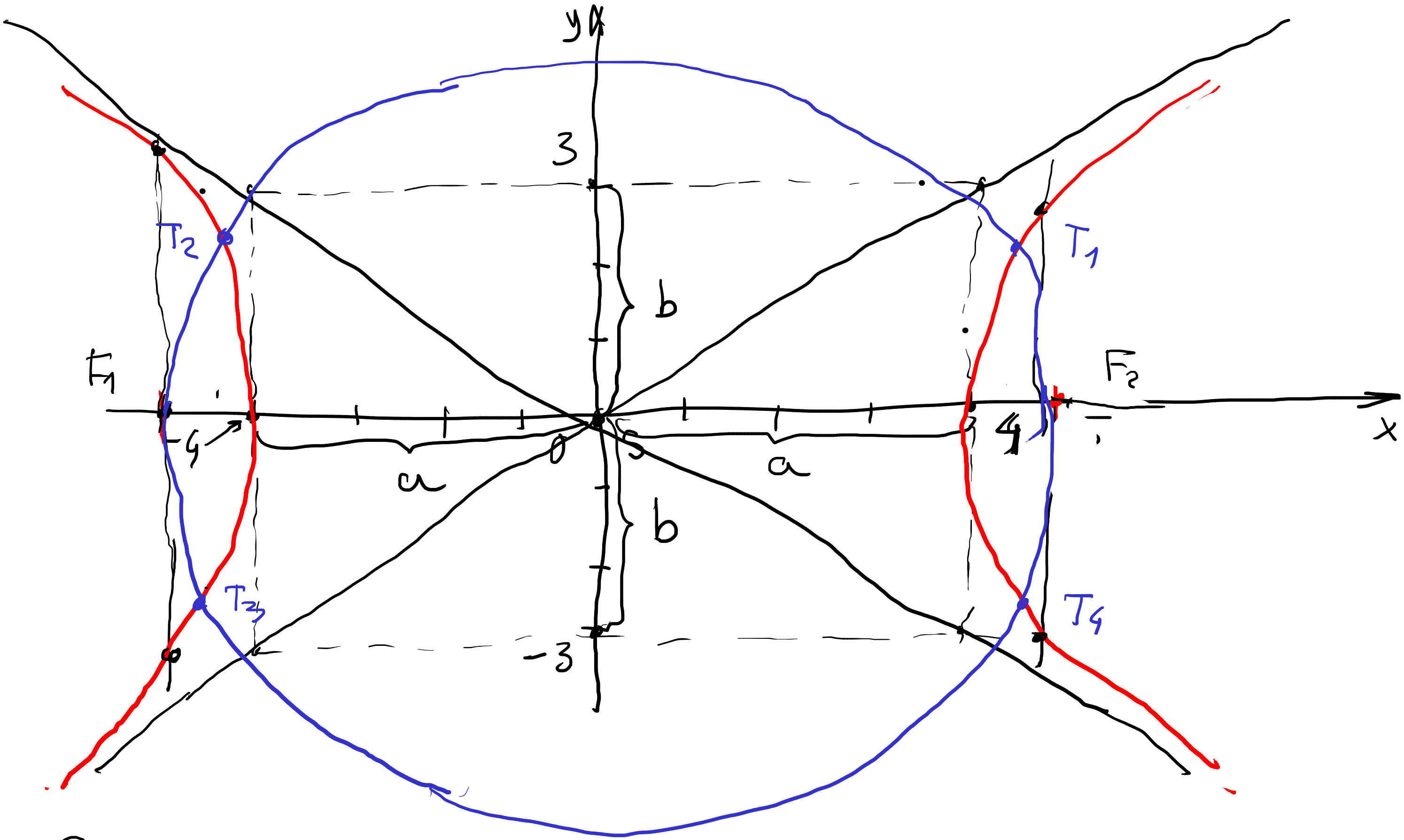
$$\frac{x^2}{16} - \frac{y^2}{9} = 1 \implies \underline{a^2 = 16, \quad a = 4} \qquad \underline{b^2 = 9, \quad b = 3}$$

žarišta hiperbole:

$e^2 = a^2 + b^2 = 16 + 9 = 25 \implies e = 5$

$F_1(-5, 0)$

$F_2(5, 0)$



(k) ... $x^2 + y^2 = e^2 = 25 \implies x^2 = 25 - y^2$

(h) ... $9x^2 - 16y^2 = 144$

$$9(25 - y^2) - 16y^2 = 144$$

⋮

$$y^2 = \frac{81}{25} \qquad y_1 = -\frac{9}{5} \qquad y_2 = \frac{9}{5}$$

$$x_1 = \frac{-4\sqrt{34}}{5} \qquad x_2 = \frac{4\sqrt{34}}{5}$$

$$\boxed{T_1\left(\frac{4\sqrt{34}}{5}, \frac{9}{5}\right); \quad T_2\left(-\frac{4\sqrt{34}}{5}, \frac{9}{5}\right); \quad T_3\left(-\frac{4\sqrt{34}}{5}, -\frac{9}{5}\right) \quad T_4\left(\frac{4\sqrt{34}}{5}, \frac{9}{5}\right)}$$

⑤ (Pr 8)

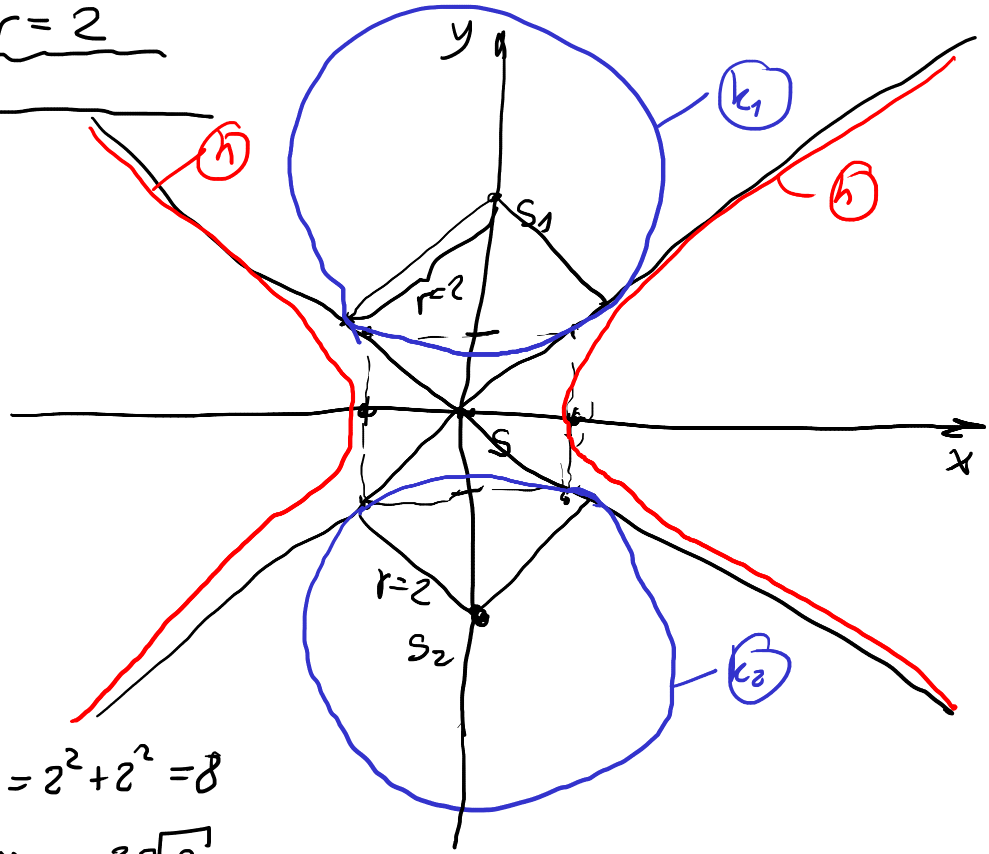
$h \dots x^2 - y^2 = 1 \Rightarrow \underline{a = b = 1}$

$P_K = 4\pi = r^2\pi \Rightarrow \underline{r = 2}$

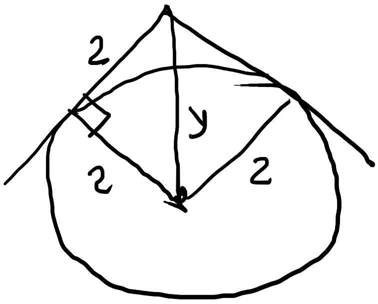
$k_1, k_2 = ?$

$S_1 \quad S_2$

$y_1 = ? \quad y_2 = ?$



$x^2 + (y - \underbrace{y_S}_{=?})^2 = 4$



$y^2 = 2^2 + 2^2 = 8$

$\Rightarrow \underline{y_1 = 2\sqrt{2}}$

$\underline{y_2 = -2\sqrt{2}}$

2 rješenja:

$$\boxed{\begin{array}{l} k_1 \dots x^2 + (y - 2\sqrt{2})^2 = 4 \\ k_2 \dots x^2 + (y + 2\sqrt{2})^2 = 4 \end{array}}$$

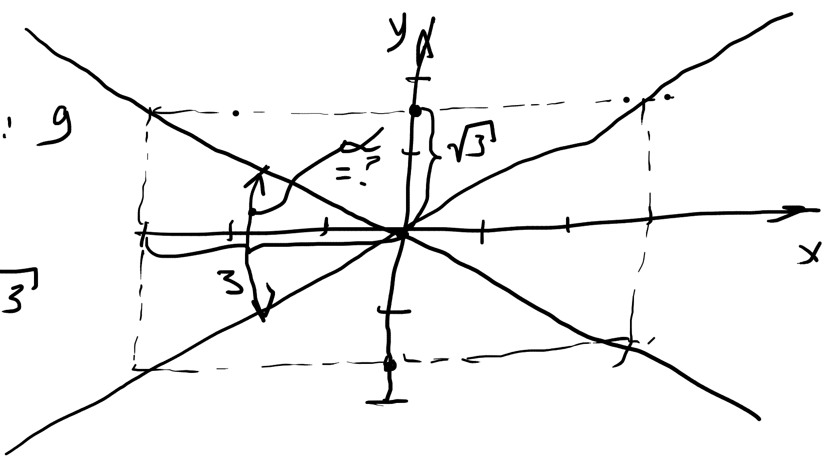
⑥ (ispit 1 - σ)

$h \dots x^2 - 3y^2 = 9 \quad /:9$

$\frac{x^2}{9} - \frac{y^2}{3} = 1$

$\hookrightarrow b = \sqrt{3}$

$\hookrightarrow a = 3$



$\alpha = ?$

$tg\left(\frac{\alpha}{2}\right) = \frac{\sqrt{3}}{3} \Rightarrow \frac{\alpha}{2} = 30^\circ$

$\boxed{\alpha = 60^\circ}$

7 (ispit 1 - 8)

$P \ldots \; y^2 = 4x \Rightarrow y_1 = 2\sqrt{x}$

$y_2 = -2\sqrt{x}$

$p \ldots \; x - 2y + 6 = 0$

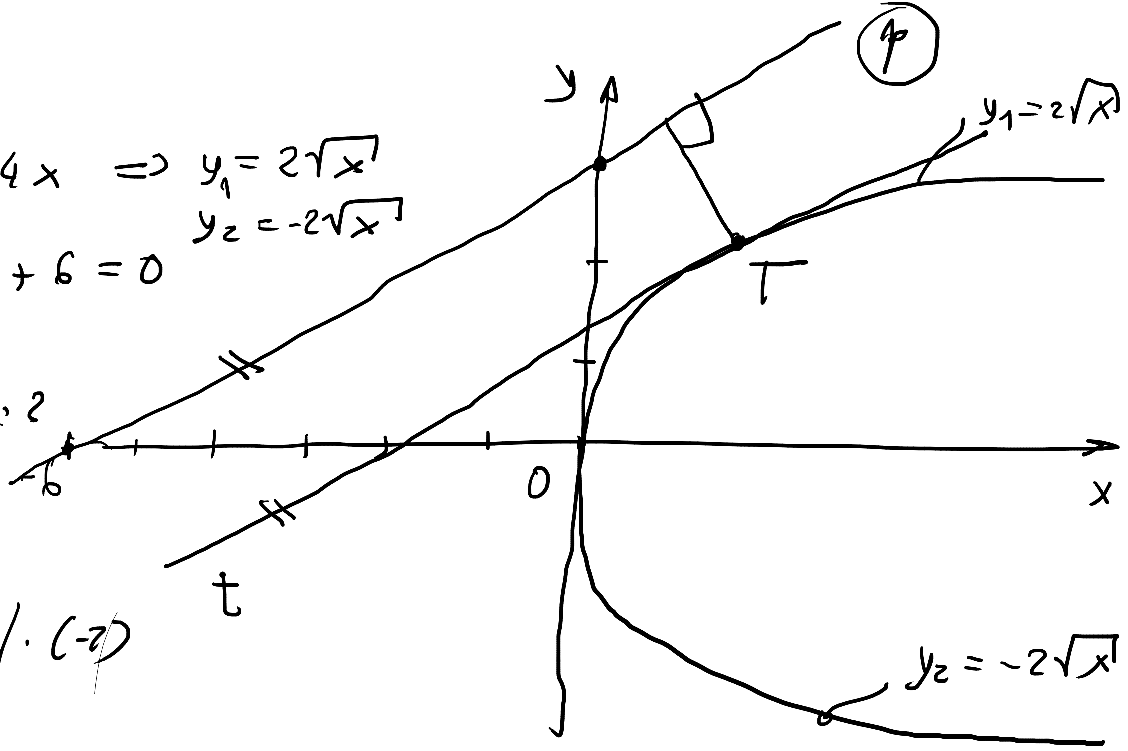
$2y = x + 6 \; / : 2$

$y = \frac{1}{2}x + 3$

$y = 0$

$-\frac{1}{2}x = 3 \; / \cdot (-2)$

$x = -6$



$t = p'$ - tangenta parabole P

$t \parallel p \Rightarrow \underline{a_t = \frac{1}{2}}$

$y_t = \frac{1}{2}x + b_t \; ?$

<u>uvjet dodira pravca i parabole:</u>

$y^2 = 4x = 2px$

$(1) \quad \Rightarrow \underline{p = 2}$

$p = 2 a_t b_t$

$2 = 2 \cdot \frac{1}{2} \cdot b_t \Rightarrow \underline{b_t = 2}$

$t \ldots \; \underline{y = \frac{1}{2}x + 2} \quad (2)$

$(1) \rightarrow (2):$

$$\left(\frac{1}{2}x + 2\right)^2 = 4x$$

$$\frac{1}{4}x^2 + 2x + 4 = 4x$$

$$\frac{1}{4}x^2 - 2x + 4 = 0 \; / \cdot 4$$

$$x^2 - \underbrace{8}_{2 \cdot 4}x + 16 = 0$$

$$(x-4)^2 = 0 \rightarrow \underline{x_{1,2} = 4} \rightarrow (2)$$

$$y = \frac{1}{2} \cdot 4 + 2 = 4$$

$$\boxed{T(4, 4)}$$

8 (Pr 11)

(p) ... $y = x^2 - 2x - 3$

$\hookrightarrow a = 1 > 0$

$\overbrace{y_0 - \text{direktrisa}}$

žarište (fokus) F : $F\left(x_0, -\frac{p}{2} + y_0\right)$

ravnalica parabole

p - parametar parabole : $a = \frac{1}{2p} \Rightarrow p = \frac{1}{2a} = \frac{1}{2}$

(d) ... $y = -\frac{p}{2} + y_0 = -\frac{1}{2} \cdot \frac{1}{2} + y_0 = -\frac{1}{4} + y_0$

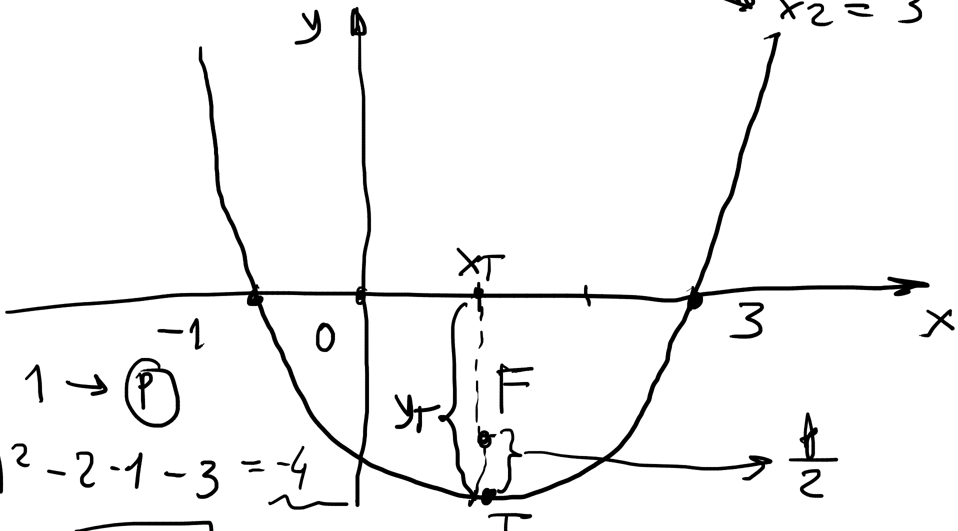
Tjeme parabole: $y = x^2 - 2x - 3$ → $x_1 = -1$, → $x_2 = 3$

$T(x_T, y_T)$

$x_T = \frac{x_1 + x_2}{2}$

$x_T = \frac{-1+3}{2} = 1 \rightarrow$ (p)

$y(x_T = 1) = 1^2 - 2 \cdot 1 - 3 = -4$



$\boxed{y} = -\frac{1}{4} - 4 = \boxed{-\frac{17}{4}}$ jednadžba direktrise (ravnalice)

$F\left(1, -\frac{17}{4} + \frac{p}{2}\right) = \boxed{F\left(1, \underbrace{-\frac{17}{4} + \frac{2}{4}}_{-\frac{15}{4}}\right)}$ fokus parabole

20. GEOMETRIJA PROSTORA

① (Pr 2)

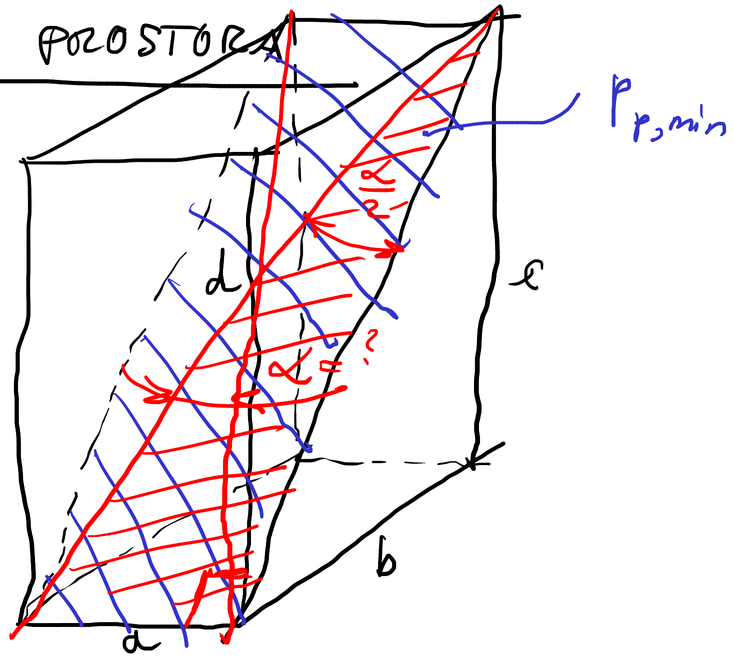
kvadar

$a = 9\text{ cm}$

$b = 12\text{ cm}$

$c = 20\text{ cm}$

najmanji dijagonalni presjek



$$\sin\left(\frac{\alpha}{2}\right) = \frac{a}{d} = \frac{9}{25}$$

$$d = \sqrt{a^2+b^2+c^2} = \sqrt{9^2+12^2+20^2} = 25\text{ cm}$$

$$\frac{\alpha}{2} = \arcsin\left(\frac{9}{25}\right) = 21{,}1^\circ \;/\cdot 2 \Rightarrow \boxed{\alpha = 42{,}2^\circ = 42^\circ 12'}$$

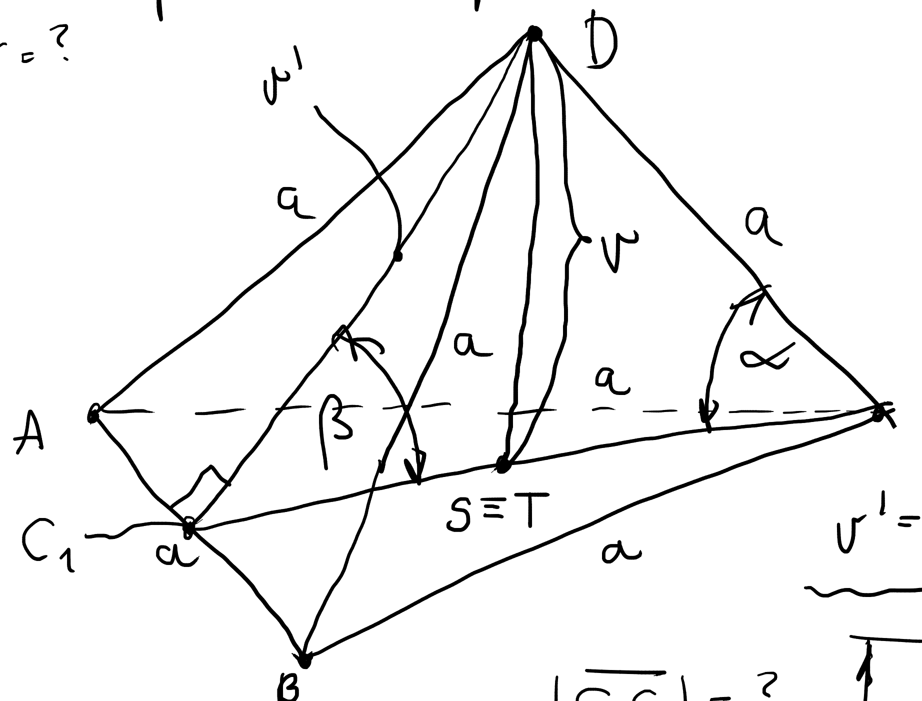
② (Pr 4)

pravilna trostrana piramida

a - duljina brida

$\beta = ?$ - priklonі kut prema ravnini osnovke piramide

$\alpha = ?$



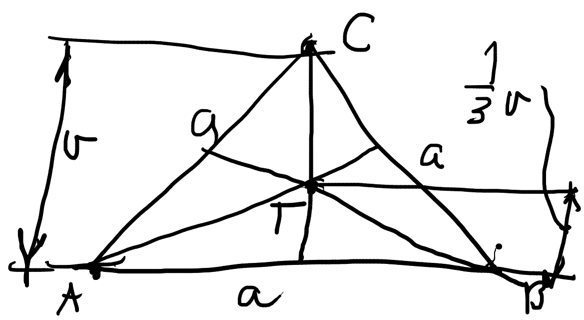
$$\sin\alpha = \frac{a}{v}$$

$$v = \sqrt{v'^2 - |\overline{C_1S}|^2}$$

ΔABD:

$$v' = \sqrt{a^2 - \frac{a^2}{4}} = \frac{\sqrt{3}}{2}a$$

$$|\overline{C_1S}| = ?$$



$$|\overline{C_1S}| = \frac{1}{3}v' = \frac{1}{3}\cdot\frac{\sqrt{3}}{2}a = \frac{\sqrt{3}}{6}a$$

$$v = \sqrt{a^2 - \left(\frac{2}{3} \cdot \frac{\sqrt{3}}{2} a\right)^2} = \frac{\sqrt{6}}{3} a$$

$$\sin\alpha = \frac{v}{a} = \frac{\frac{\sqrt{6}}{3}\cancel{a}^1}{\cancel{a}_1} = \frac{\sqrt{6}}{3} \Rightarrow \boxed{\alpha = 54{,}736^\circ = 54^\circ 44'}$$

$$\sin\beta = \frac{v}{v'} = \frac{\frac{\sqrt{6}}{3} a}{\frac{\sqrt{3}}{2} a} = \frac{2\sqrt{2}}{3}$$

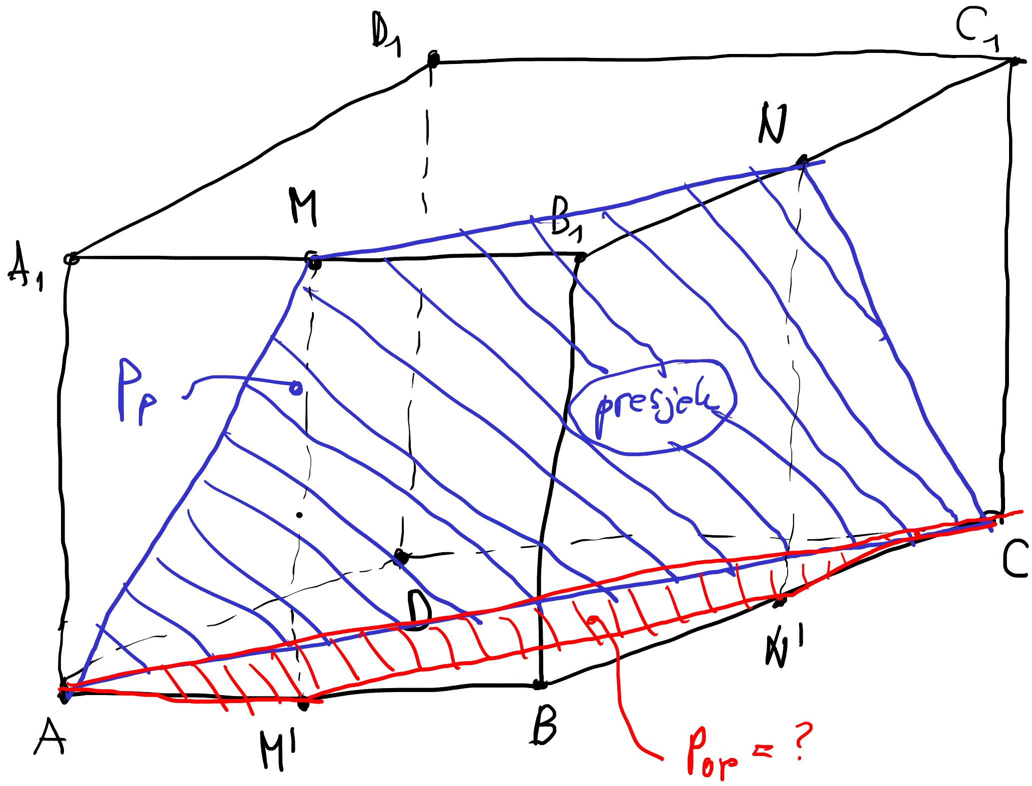
$$\Rightarrow \boxed{\beta = 70{,}529^\circ = 70^\circ 31'}$$

③ (P, 5)

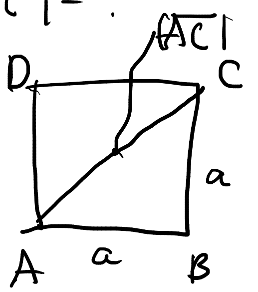
kocka

$P_p = ?$

$P_{op} = ?$

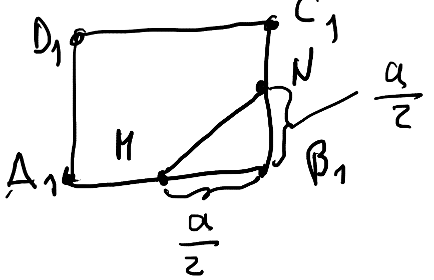


$|AC| = ?$



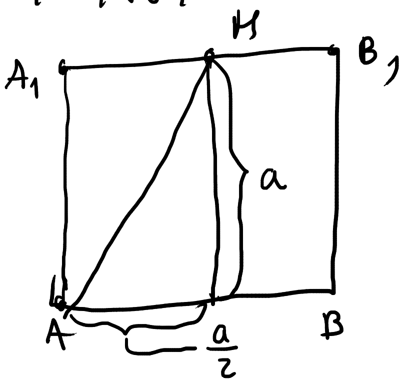
$$|AC| = \sqrt{a^2 + a^2} = \sqrt{2}\, a$$

$|MN| = ?$



$$|MN| = \sqrt{\frac{a^2}{4} + \frac{a^2}{4}} = \frac{\sqrt{2}}{2} a$$

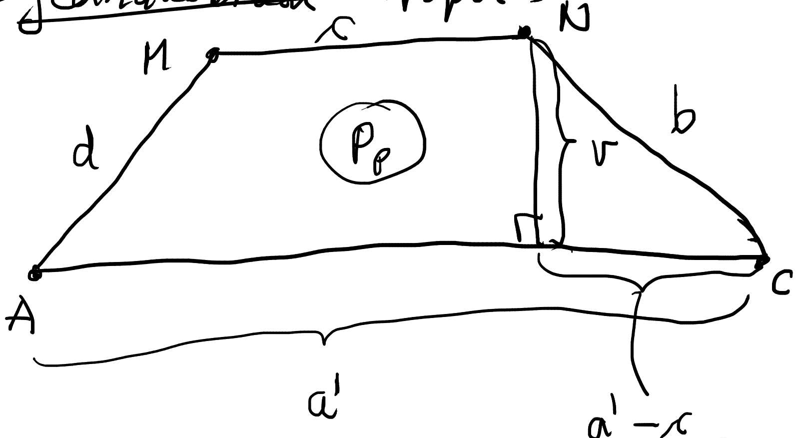
$|AM| = |NC| = ?$



$$|AM| = |NC| = \sqrt{\frac{a^2}{4} + a^2} = \frac{\sqrt{5}}{2} a$$

Presječna ravnina

- jednakokračni trapez



$$\frac{a'-c}{2} = \frac{\sqrt{2}}{4}a$$

$$P_t = \frac{a'+c}{2}\cdot v$$

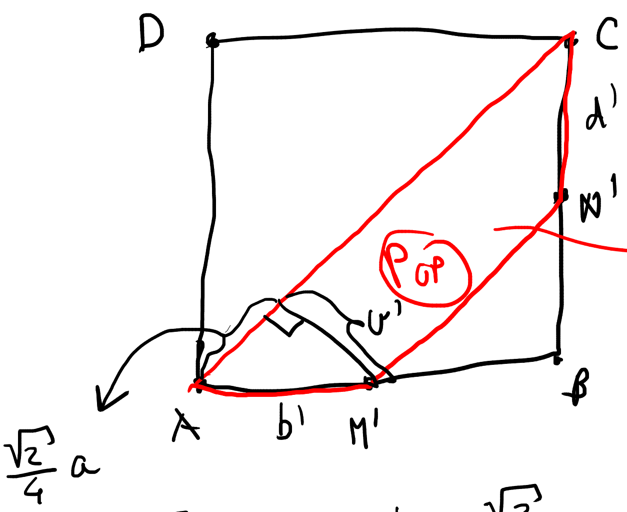
$$v = \sqrt{b^2 - \left(\frac{a'-c}{2}\right)^2}$$

$$v = \sqrt{\left(\frac{\sqrt{5}}{2}a\right)^2 - \left(\frac{\sqrt{2}}{4}a\right)^2}$$

$$v = \frac{3\sqrt{2}}{4}a$$

$$\boxed{P_p = \frac{\sqrt{2}a + \frac{\sqrt{2}}{2}a}{2}\cdot\frac{3\sqrt{2}}{4}a = \frac{9}{8}a^2}$$

Površina ort. projekcije na osnovku - ravnim ABCD:



jednakokračni trapez

$$P_{op} = \frac{|\overline{AC}| + |\overline{M'N'}|}{2}\cdot v'$$

$$|\overline{M'N'}| = |\overline{MN}| = \frac{\sqrt{2}}{2}a$$

$$|\overline{AC}| = \sqrt{2}a$$

$$b' = d' = |\overline{AM'}| = |\overline{CN'}| = \frac{a}{2}$$

$$v' = \sqrt{\left(\frac{a}{2}\right)^2 - \left(\frac{\sqrt{2}}{4}a\right)^2} = \frac{\sqrt{2}}{4}a$$

$$P_{op} = \frac{\sqrt{2}a + \frac{\sqrt{2}}{2}a}{2}\cdot\frac{\sqrt{2}}{4}a$$

$$\boxed{P_{op} = \frac{3}{8}a^2}$$

④ (Pr 7)

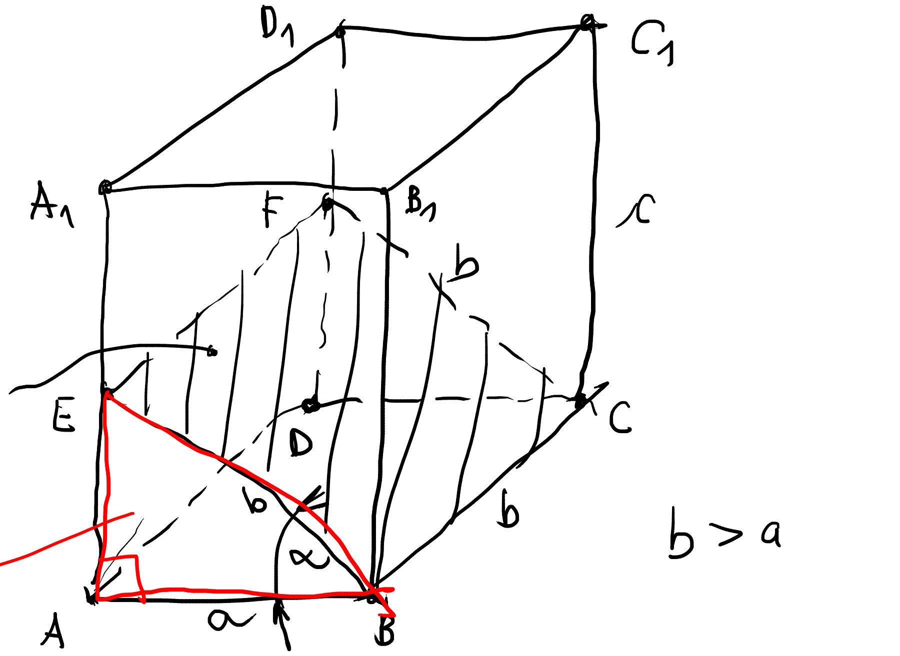
kvadar

$a = 4\ cm$

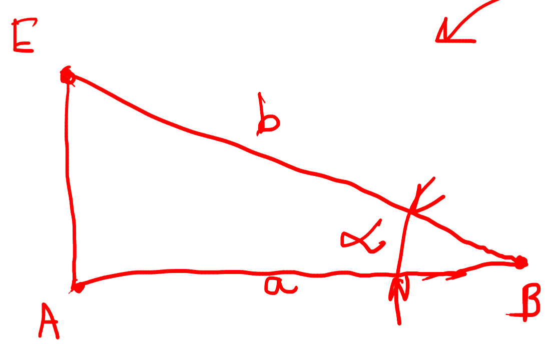
$b = 5\ cm$

$c = 6\ cm$

presjek – kvadrat $b \times b$



$b > a$



$$\cos\alpha = \frac{a}{b} = \frac{4}{5} = 0{,}8$$

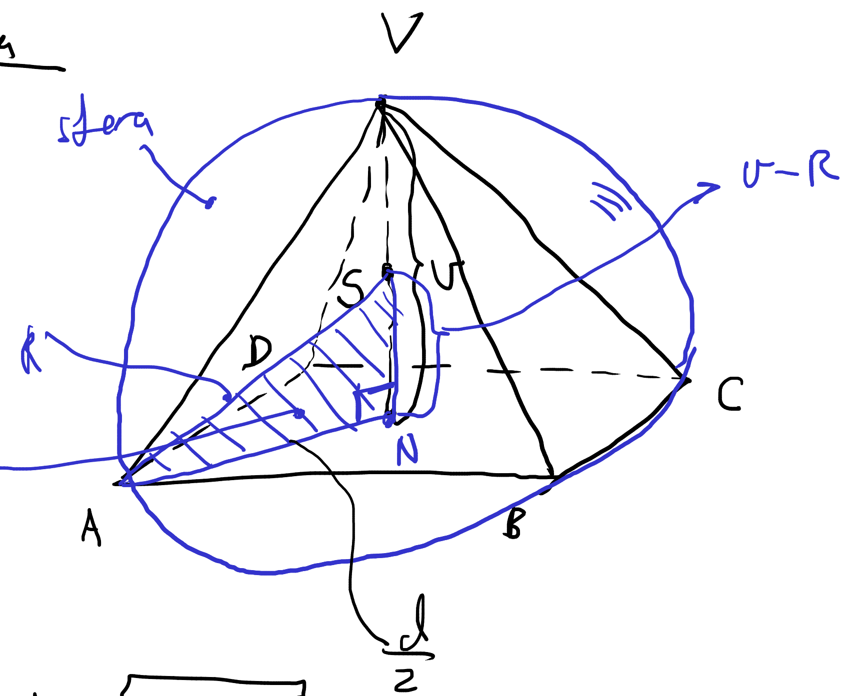
$$\Rightarrow \boxed{\alpha = \arccos(0{,}8) = 36{,}869^\circ = 36^\circ 52' 11''}$$

⑤ ~~četverostrana piramida~~

$a = 8\ cm$

$v = 10\ cm$

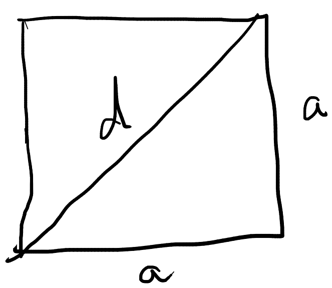
$R = ?$



trokut SAN:

$$R^2 = (\overset{?}{v} - R)^2 + \left(\frac{d}{2}\right)^2 \quad = f(a)$$

$a = ?$



$$d = \sqrt{a^2 + a^2}$$

$$d = \sqrt{2}\,a$$

$$\frac{d}{2} = \frac{\sqrt{2}}{2}\,a$$

$$\cancel{R^2} = v^2 - 2vR + \cancel{R^2} + \left(\frac{\sqrt{2}}{2}a\right)^2$$

$$\Rightarrow \boxed{R = \frac{v^2 + \frac{1}{2}a^2}{2v} = \frac{10^2 + \frac{1}{2}\cdot 8^2}{2\cdot 10} = 6{,}6\ cm}$$

21. VEKTORI

① kvadrat ABCD
S – sjecište dijagonala

$\vec{AM} = \vec{m}$
$\vec{AN} = \vec{n}$

$\vec{AC} = ?$, $\vec{BD} = ?$, $\vec{AB} = ?$, $\vec{BC} = ?$

$\vec{AC} = ?$
∟ pravila trokuta

$$\boxed{\vec{AC} = \vec{m} + \vec{n}}$$

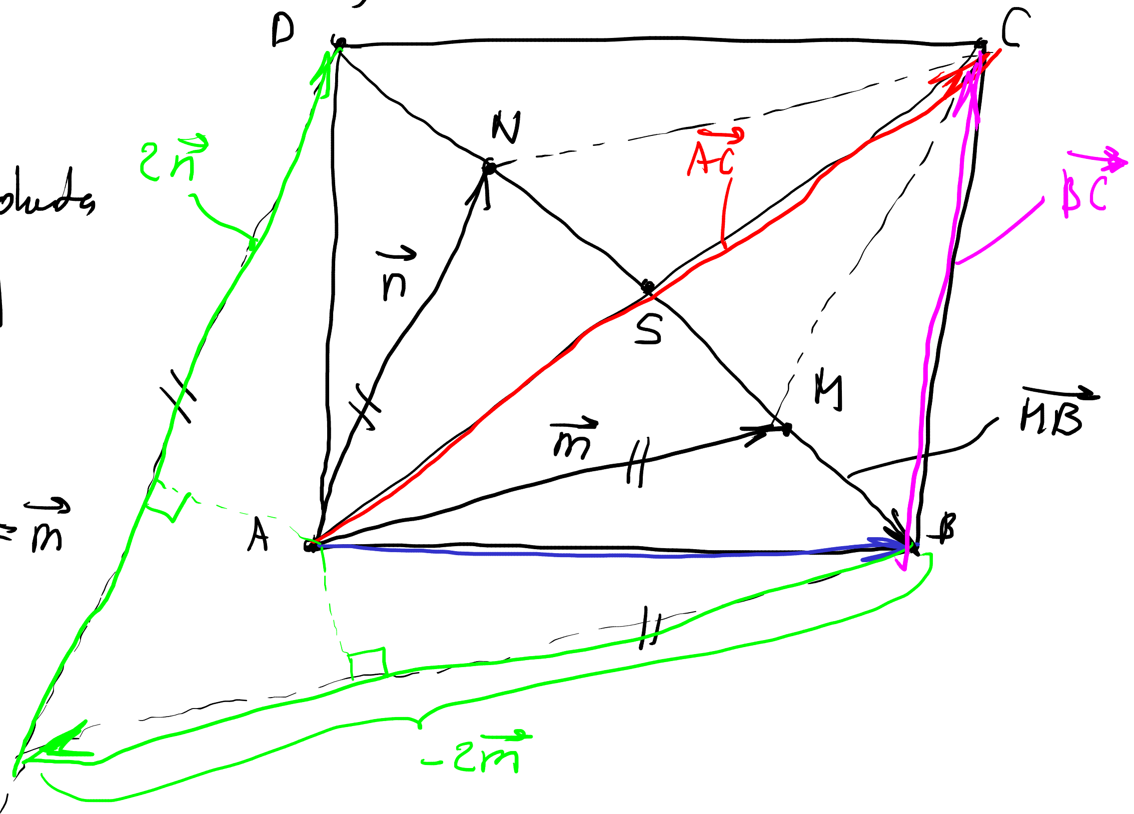
$\vec{BD} = \ldots$

$\vec{AB} = \vec{AM} + \vec{MB} = \vec{m}$

$\vec{MB} = \frac{1}{4}\vec{BD}$

$\vec{MB} = -\frac{1}{4}\cdot(-\cancel{2}\vec{m} + \cancel{2}\vec{n})$

$= -\frac{1}{2}(-\vec{m} + \vec{n}) = \frac{1}{2}\vec{m} - \frac{1}{2}\vec{n}$



$$\boxed{\vec{AB} = \frac{3}{2}\vec{m} - \frac{1}{2}\vec{n}}$$

$$\boxed{\vec{BC} = -\vec{AB} + \vec{AC} = -\left(\frac{3}{2}\vec{m} - \frac{1}{2}\vec{n}\right) + (\vec{m} + \vec{n}) = -\frac{1}{2}\vec{m} + \frac{3}{2}\vec{n}}$$

$$\vec{BD} = -\vec{AB} + \vec{BC} = -\left(\frac{3}{2}\vec{m} - \frac{1}{2}\vec{n}\right) + \left(-\frac{1}{2}\vec{m} + \frac{3}{2}\vec{n}\right)$$

$$= \boxed{-2\vec{m} + 2\vec{n}}$$

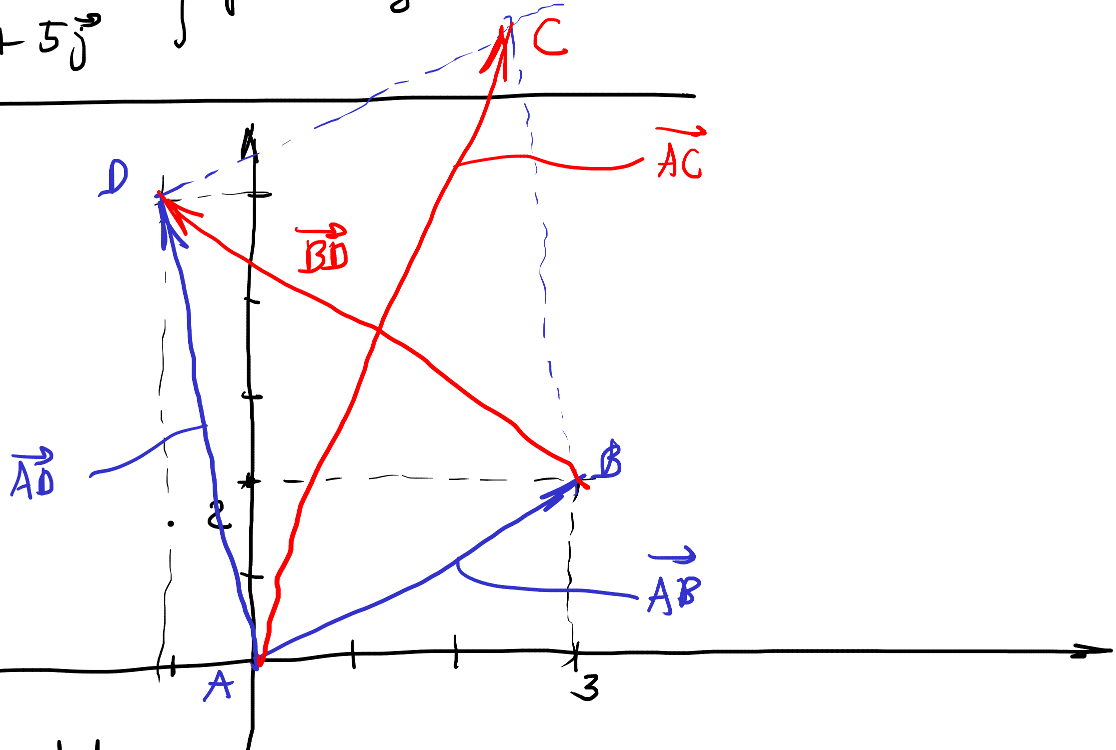
② (Pr 3)

$$\left.\begin{aligned} \vec{AB} &= 3\vec{i} + 2\vec{j} \\ \vec{AD} &= -\vec{i} + 5\vec{j} \end{aligned}\right\}$$ paralelogram ABCD

a) $|\vec{AC}| = ?$

$|\vec{BD}| = ?$

b) $\varphi = ?$



a) Dijagonale paralelograma:

$$\vec{AC} = \vec{AB} + \vec{AD} = 3\vec{i} + 2\vec{j} + (-\vec{i} + 5\vec{j}) = 2\vec{i} + 7\vec{j}$$

$$\boxed{|\vec{AC}| = \sqrt{2^2 + 7^2} = \sqrt{53}}$$

$$\vec{BD} = -\vec{AB} + \vec{AD} = -(3\vec{i} + 2\vec{j}) + (-\vec{i} + 5\vec{j}) = -4\vec{i} + 3\vec{j}$$

$$\boxed{|\vec{BD}| = \sqrt{(-4)^2 + 3^2} = 5}$$

b) Kut između dijagonala paralelograma φ:

skalarni umnožak vektora

$$\vec{AC} \cdot \vec{BD} = |\vec{AC}| \cdot |\vec{BD}| \cdot \cos\varphi$$

$$\Rightarrow \cos\varphi = \frac{\vec{AC} \cdot \vec{BD}}{|\vec{AC}| \cdot |\vec{BD}|} = \frac{2 \cdot (-4) + 7 \cdot 3}{\sqrt{53} \cdot 5} = \frac{13}{5\sqrt{53}}$$

$$\Rightarrow \boxed{\varphi = 69^\circ\, 4'\, 32''}$$

(3)

(Pr 4)

$$\vec{a} = -\vec{i} - \vec{j}$$
$$\left.\begin{aligned}\vec{b} &= \vec{i} + 3\vec{j} \\ \vec{c} &= -2\vec{i} + 4\vec{j}\end{aligned}\right\} \quad \begin{aligned}\vec{b}+\vec{c} &= -\vec{i} + 7\vec{j} \\ |\vec{b}+\vec{c}| &= \sqrt{(-1)^2+7^2} = \sqrt{50} = 5\sqrt{2}\end{aligned}$$

$\vec{v} = ?$ — kolinearan s vektorom $\vec{a}$
duljina od $\vec{v} = |\vec{b} + \vec{c}|$

kolinearnost vektora $\quad \vec{v} = k \cdot \vec{a}, \; k \neq 0, \; k \in \mathbb{R}$

$$k \cdot \vec{a} = -k\vec{i} - k\vec{j}$$
$$|\vec{v}| = \sqrt{(-k)^2 + (-k)^2} = \sqrt{2k^2} = \sqrt{2}\,|k|$$

$$|\vec{v}| = |\vec{b} + \vec{c}|$$
$$|k|\sqrt{2} = 5\sqrt{2}$$
$$|k| = 5 \implies k_{1,2} = \pm 5$$

$$\boxed{\vec{v} = \pm 5(-\vec{i} - \vec{j})}$$

(4) (zadni – 11)

$$\vec{a} = -2\vec{i} + \vec{j}$$
$$\vec{b} = 3\vec{i} - 2\vec{j}$$

$$\vec{a} \cdot \vec{c} = 3$$
$$\vec{b} \cdot \vec{c} = -5$$

$$\left.\begin{aligned}\vec{a} \cdot \vec{b} &= a_x c_x + a_y c_y = -2c_x + c_y = 3 \;/\cdot 2 \\ \vec{b} \cdot \vec{c} &= b_x c_x + b_y c_y = 3c_x - 2c_y = -5\end{aligned}\right\} (+)$$

$$\left.\begin{aligned}-4c_x + \cancel{2c_y} &= 6 \\ 3c_x - \cancel{2c_y} &= -5\end{aligned}\right\} (+)$$

$$-c_x = 1$$

$$c_x = -1 \qquad c_y = 1$$

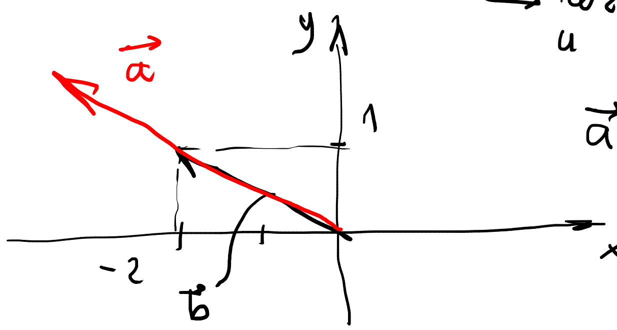
$$\boxed{\vec{c} = -\vec{i} + \vec{j}}$$

(5) (zadaci - 13)

$\vec{b} = -2\vec{i} + \vec{j}$ $\qquad |\vec{b}| = \sqrt{(-2)^2 + 1^2} = \sqrt{5}$

$\vec{a} = ?$ - isti smjer kao $\vec{b}$, ista orijentacija $\qquad |\vec{a}| = 3\sqrt{5}$

→ leže na paralelnim pravcima i gledaju u istu stranu



$$\vec{a}\cdot\vec{b} = |\vec{a}|\cdot|\vec{b}|\cdot\underbrace{\cos \underbrace{\varphi}_{=0}}_{=1}$$

$$a_x b_x + a_y b_y = \sqrt{5}\cdot 3\sqrt{5}$$

$$-2a_x + a_y = 15 \Rightarrow a_y = 15 + 2a_x$$

$$a_x^2 + a_y^2 = (3\sqrt{5})^2 = 45$$

$$a_x^2 + (15 + 2a_x)^2 = 45$$

$$a_x^2 + 225 + 60a_x + 4a_x^2 = 45$$

$$5a_x^2 + 60a_x + 180 = 0 \quad /:5$$

$$a_x^2 + 12a_x + 36 = 0$$

$$(a_x - 6)^2 = 0$$

$(a_x)_1 = -6$, $(a_x)_2 = -6$ $\Big\}$ $a_{x_{1,2}} = -6$

$$a_y = 15 + 2\cdot(-6) = 3$$

$$\boxed{\vec{a} = -6\vec{i} + 3\vec{j}}$$

(6) (zadaci - 16)

jedinični vektor - okomit na vektor $\overrightarrow{AB}$

$A(-1, 2)$

$B(3, -1)$

$\vec{e} = e_x \vec{i} + e_y \vec{j}$

$|\vec{e}| = 1 = \sqrt{e_x^2 + e_y^2} \quad (1) \; /^2$

$\vec{AB} \perp \vec{e} \qquad \vec{AB} \cdot \vec{e} = \vec{0}$

$a_x e_x + a_y e_y = 0$

$\vec{AB} = \sqrt{(3-(-1))^2}\,\vec{i} + \sqrt{(-1-2)^2}\,\vec{j} = 4\vec{i} + 3\vec{j} = \vec{a}$

$4e_x + 3e_y = 0 \quad (2)$

$e_x^2 + e_y^2 = 1$

iz (2) : $\underline{e_x = -\frac{3}{4} e_y}$

$\left(-\frac{3}{4} e_y\right)^2 + e_y^2 = 1$

$\frac{9}{16} e_y^2 + e_y^2 = 1$

$\frac{25}{16} e_y^2 = 1 \; / \cdot \frac{16}{25} \Rightarrow \underline{e_y = \frac{4}{5}}$

$\underline{e_x} = -\frac{3}{4} \cdot \frac{4}{5} = \underline{-\frac{3}{5}}$

$\boxed{\vec{e} = -\frac{3}{5}\vec{i} + \frac{4}{5}\vec{j}}$

(7) (ispit 1−6)

$$\vec{AB} = -\vec{i} + 2\vec{j} = \vec{a}$$

$$\vec{CD} = 3\vec{i} - \vec{j} = \vec{b}$$

Kut među vektorima:

$$\cos\varphi = \frac{a_x b_x + a_y b_y}{\sqrt{a_x^2 + a_y^2} \cdot \sqrt{b_x^2 + b_y^2}} = \frac{-1 \cdot 3 + 2 \cdot (-1)}{\sqrt{(-1)^2 + 2^2} \cdot \sqrt{3^2 + (-1)^2}}$$

$$= \frac{-3-2}{\sqrt{5} \cdot \sqrt{10}} = \frac{-5}{\sqrt{50}} = -\frac{5}{5\sqrt{2}} = -\frac{\sqrt{2}}{2}$$

$$\Rightarrow \boxed{\varphi = \arccos\left(-\frac{\sqrt{2}}{2}\right) = 135^\circ = \frac{3\pi}{4}}$$

(8) (ispit 1−8)

A(−2, 1)

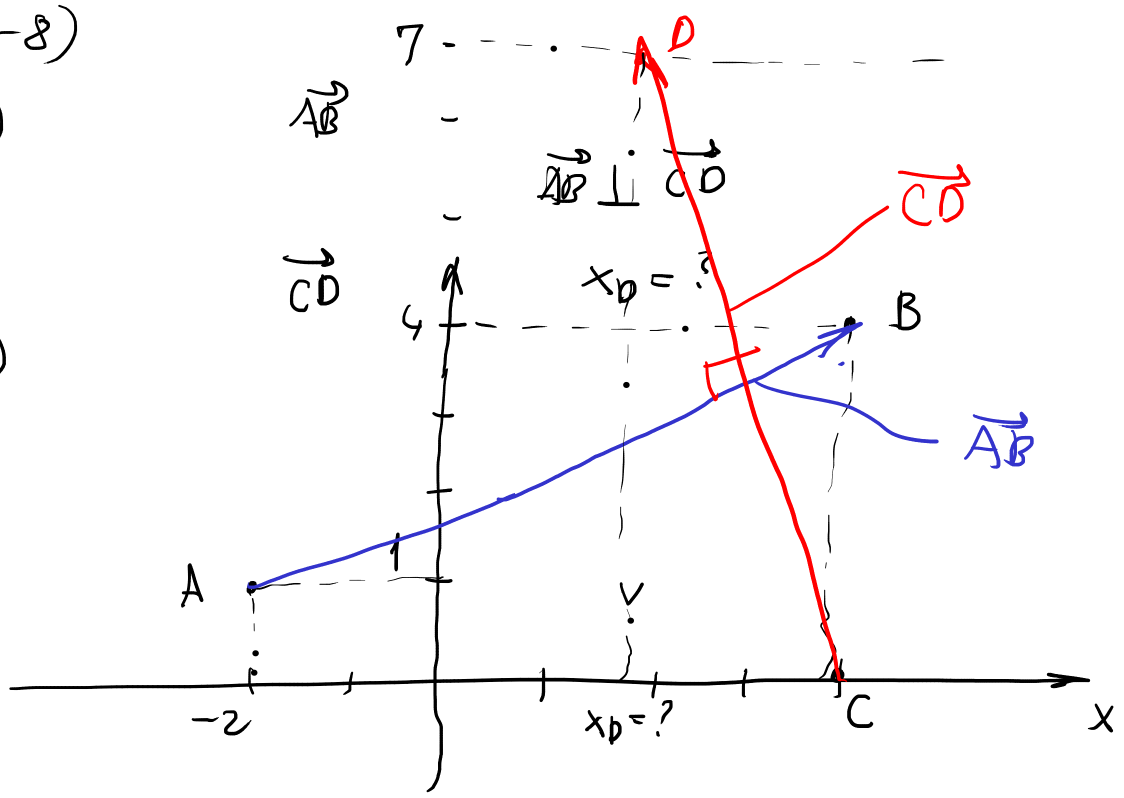
B(4, 4)

C(4, 0)

D(x, 7)

$\vec{AB}$

$\vec{CD}$



$$\cos 90^\circ = 0$$

$$a_x b_x + a_y b_y = 0$$

$$\cos\varphi = \frac{a_x b_x + a_y b_y}{\sqrt{a_x^2 + a_y^2} \cdot \sqrt{b_x^2 + b_y^2}} = 0$$

$\vec{a} = \overrightarrow{AB}$ ..... $(x_B - x_A)\vec{i} + (y_B - y_A)\vec{j} = \vec{0}$

$$(4-(-2))\vec{i} + (4-1)\vec{j} = \vec{0}$$

$$6\vec{i} + 3\vec{j} = \vec{0}$$

$\vec{b} = \overrightarrow{CD}$ ..... $(x_D - x_C)\vec{i} + (y_D - y_C)\vec{j} = \vec{0}$

$$(x_D - 4)\vec{i} + (7-0)\vec{j} = \vec{0}$$

$$(x_D - 4)\vec{i} + 7\vec{j} = \vec{0}$$

$\vec{a} \cdot \vec{b} = \vec{0}$ .... $6 \cdot (x_D - 4) + 3 \cdot 7 = 0$

$$6x_D - 24 + 21 = 0$$

$$6x_D - 3 = 0$$

$$\rightarrow \boxed{x_D = \frac{1}{2}}$$