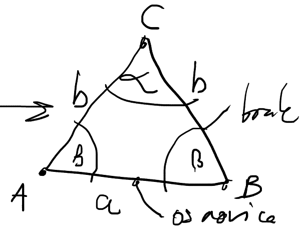
13. TRIGONOMETRIJA PRAVOKUTNOG TROKUTA

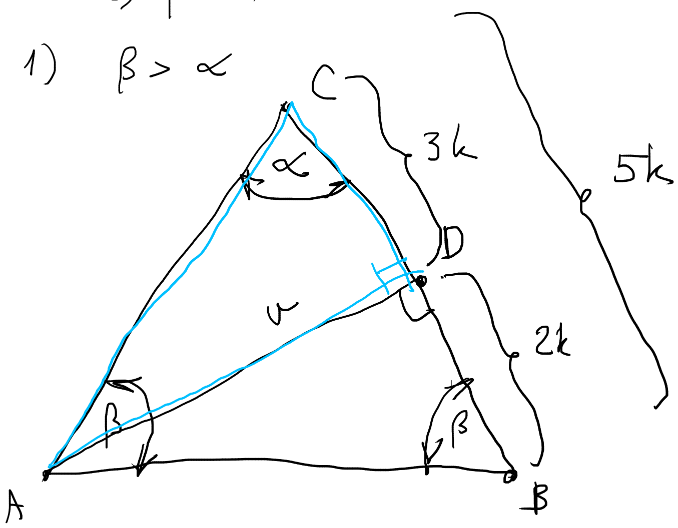
① (Pr 3)

jednakokračni trokut



$\alpha, \beta = ?$

1) $\beta > \alpha$



2) $\alpha > \beta$

$$\frac{|\overline{CD}|}{|\overline{BD}|} = \frac{3}{2}$$

$$\frac{|\overline{CD}|}{|\overline{BC}|} = \frac{3k}{5k} = \frac{3}{5} = 0{,}6$$

ΔADC – pravokutni trokut

$$\frac{|\overline{CD}|}{|\overline{AC}|} = \cos\alpha = 0{,}6$$

$$\Rightarrow \boxed{\alpha = \arccos(0{,}6) = 53{,}13^\circ = 53^\circ 8'}$$

$$\alpha + 2\beta = 180^\circ$$

$$\Rightarrow \beta = \frac{180^\circ - \alpha}{2} = 90^\circ - \frac{1}{2}\alpha$$

$$= 90^\circ - \frac{1}{2}\cdot 53{,}13^\circ = \boxed{63{,}44^\circ = 63^\circ 26'}$$

2) $\alpha > \beta \Rightarrow \boxed{\begin{array}{l}\alpha = 66^\circ 25' \\ \beta = 56^\circ 47'\end{array}}$

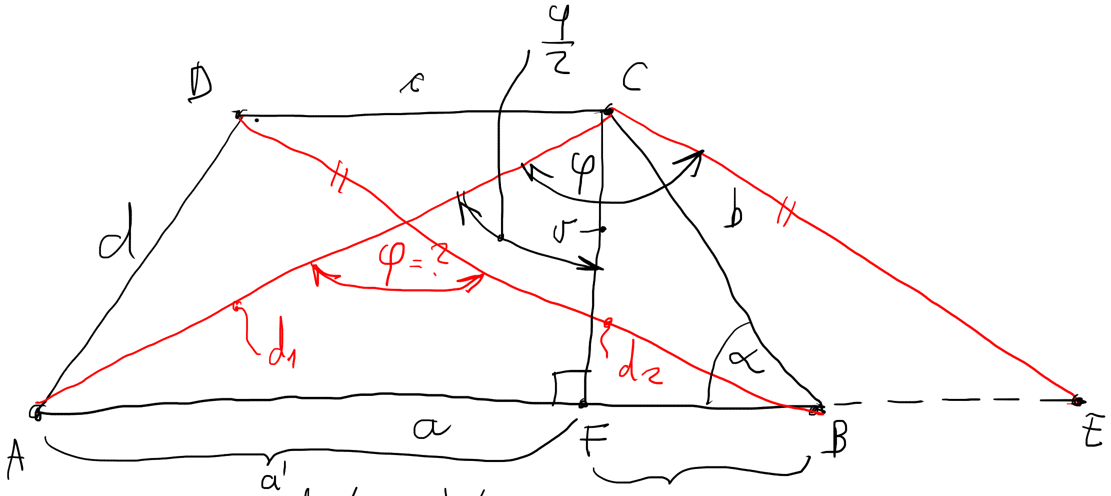
② jednakokračni trapez → ima 2 kraka jednake duljine b, d

$|\overline{AB}| = a = 11$ cm

$|\overline{CD}| = c = 3$ cm

$b = d = 7$ cm

$\varphi = ?$ – kut koji zatvaraju dijagonale trapeza



ΔAFC – pravokutni trokut $\qquad \frac{a-c}{2}$

$$tg\left(\frac{\varphi}{2}\right) = \frac{a'}{v}$$

$$a' = a - \frac{a-c}{2} = 11 - \frac{11-3}{2} = 11 - 4 = 7\ cm$$

$v = ?$

ΔFBC – pravokutni trokut

$$\frac{v}{b} = \sin \alpha\ ?$$

$$\frac{\frac{a-c}{2}}{b} = \cos\alpha = \frac{\frac{11-3}{2}}{7} = \frac{4}{7} \Rightarrow \alpha = \arccos\left(\frac{4}{7}\right) = 55^\circ 9'$$

$55,15^\circ =$

$$\Rightarrow v = b\sin\alpha = 7 \cdot \sin(55,15^\circ) \approx 5,7\ cm$$

$$tg\left(\frac{\varphi}{2}\right) = \frac{7}{5,7} \approx 1,23 \Rightarrow \frac{\varphi}{2} = arctg\,(1,23) = 50,845^\circ \ / \cdot 2$$

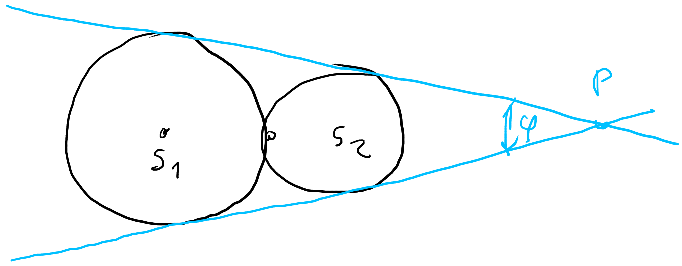
$$\boxed{\varphi = 2 \cdot 50,845^\circ = 101,69^\circ = 101^\circ 41'}$$

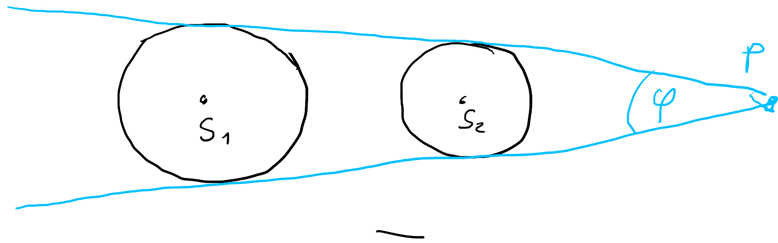
3. kružnice

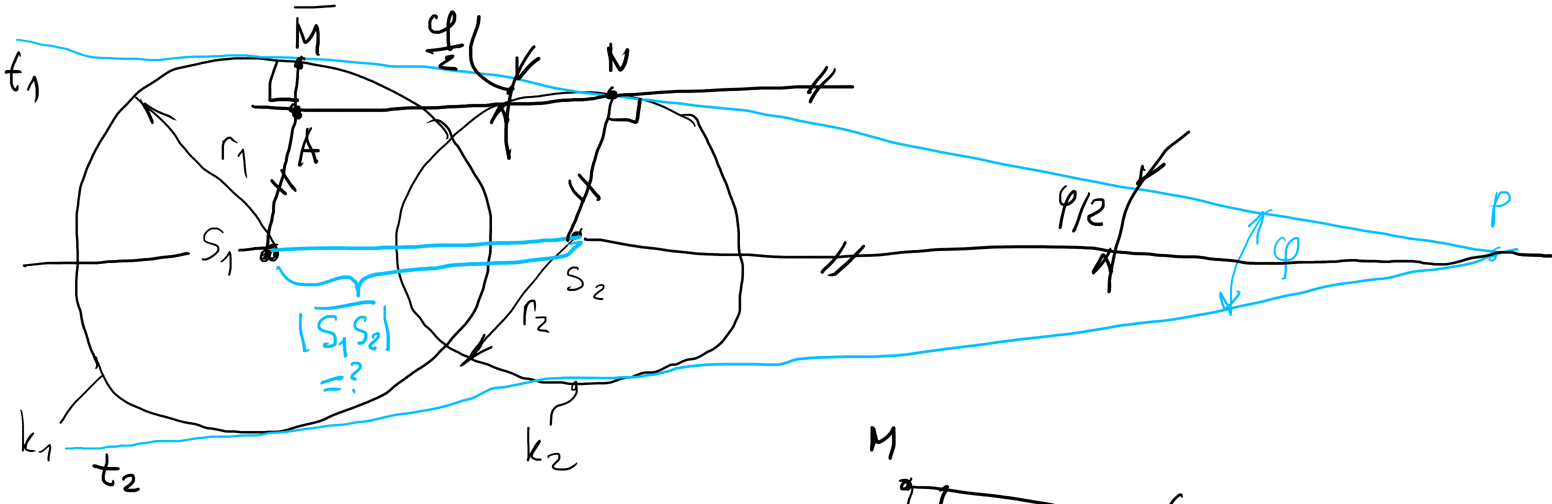
1. ... $r_1 = 10\ cm$
2. ... $r_2 = 6\ cm$

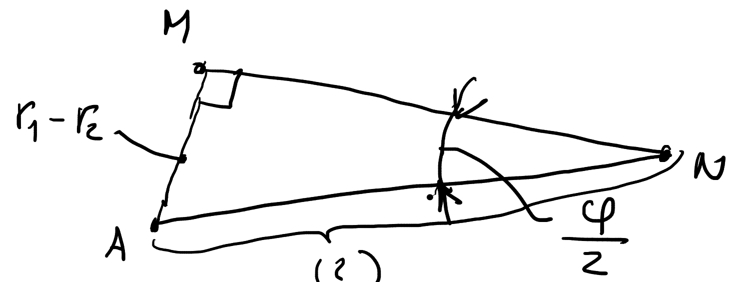
tangente kruž. 1 i 2 sijeku se pod kutem $\varphi = 42^\circ 45'$

$|S_1S_2| = ?$









ΔANM – pravoúhlý trojúhelník

$|AN| = |S_1S_2|$

$$\frac{|AM|}{|AN|} = \sin\left(\frac{\varphi}{2}\right) \Rightarrow |AN| = \frac{|AM|}{\sin\left(\frac{\varphi}{2}\right)} = \frac{10-6}{\sin\left(\frac{42{,}75°}{2}\right)} \boxed{\approx 10{,}97\ cm}$$

$$\boxed{= |S_1S_2|}$$

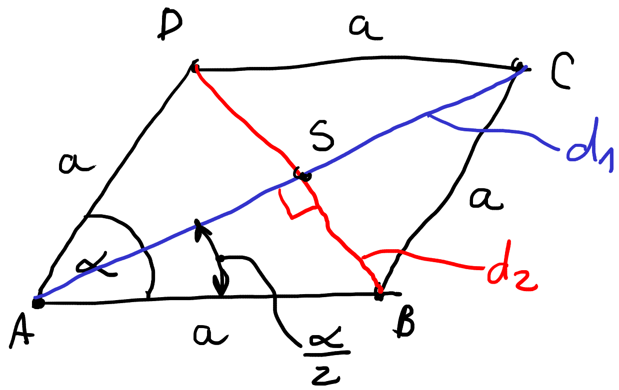
④ (Př 9)

romb

$d_1 + d_2 = 28\ cm$

$O = 40\ cm$

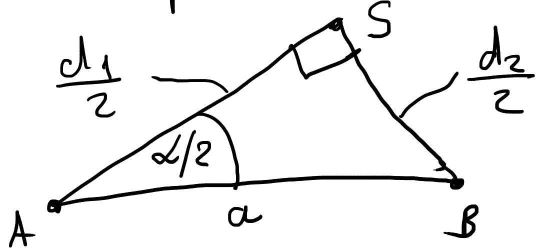
$\alpha = ?$



$\Rightarrow d_1 = 28 - d_2 \quad (1)$

$O = 4a \Rightarrow a = \frac{O}{4} = \frac{40}{4} = 10\ cm$

ΔABS – pravoúhlý trojúhelník



$$tg\left(\frac{\alpha}{2}\right) = \frac{\frac{d_2}{\cancel{2}_1}}{\frac{d_1}{\cancel{2}_1}} = \frac{d_2}{d_1}$$

Pitagorin teorem:

$$\left(\frac{d_1}{2}\right)^2 + \left(\frac{d_2}{2}\right)^2 = a^2 \quad ✓$$

(1)

$$\frac{d_1^2}{4} + \frac{d_2^2}{4} = 100 \;/\cdot 4$$

$d_1 \to$ $d_1^2 + d_2^2 = 400$

$$(28 - d_2)^2 + d_2^2 = 400 \qquad d_2 < d_1$$

$\vdots$

$$d_2^2 - 28 d_2 + 192 = 0$$

$\to (d_2)_1 = 12 \text{ cm}$ ✓

$\to (d_2)_2 = 16 \text{ cm}$

$$\boxed{\begin{aligned} d_2 &= 12 \text{ cm} \\ d_1 &= 16 \text{ cm} \end{aligned}} \Rightarrow tg\left(\frac{\alpha}{2}\right) = \frac{12}{16} = 0{,}75$$

$$\Rightarrow \frac{\alpha}{2} = 36{,}87^\circ$$

$$\boxed{\alpha = 73{,}74^\circ = 73^\circ 44'}$$

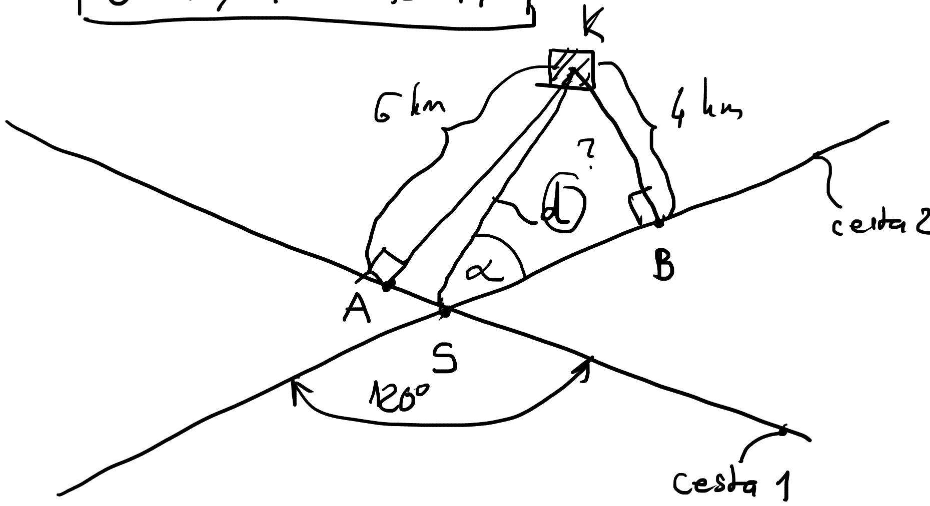
(5) dvije ravne ceste

120°

6 km

4 km

$d = ?$



dva pravokutna trokuta ΔAKS i ΔBKS

$$\left.\begin{aligned} \frac{4}{d} &= \sin\alpha \quad (*) \\ \frac{6}{d} &= \sin(120^\circ - \alpha) \end{aligned}\right\} \quad \frac{4}{\sin\alpha} = \frac{6}{\underbrace{\sin(120^\circ - \alpha)}}$$

$$= \underbrace{\sin 120^\circ}_{\frac{\sqrt{3}}{2}} \cos\alpha - \sin\alpha \cdot \underbrace{\cos 120^\circ}_{-\frac{1}{2}}$$

$$= \frac{\sqrt{3}}{2} \cos\alpha + \frac{1}{2} \sin\alpha$$

$$\frac{4}{\sin\alpha} = \frac{6}{\frac{\sqrt{3}}{2}\cos\alpha + \frac{1}{2}\sin\alpha}$$

$$6\sin\alpha = 4\left(\frac{\sqrt{3}}{2}\cos\alpha + \frac{1}{2}\sin\alpha\right)$$

$$6\sin\alpha = 2\sqrt{3}\cos\alpha + 2\sin\alpha \quad /:2$$

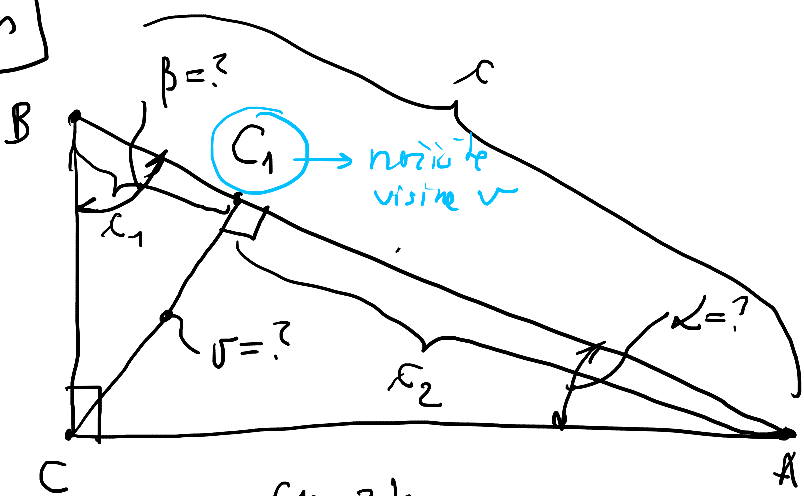
$$3\sin\alpha = \sqrt{3}\cos\alpha + \sin\alpha$$

$$2\sin\alpha - \sqrt{3}\cos\alpha = 0 \quad /:\cos\alpha$$

$$2\,\mathrm{tg}\,\alpha = \sqrt{3} \quad /:2$$

$$\mathrm{tg}\,\alpha = \frac{\sqrt{3}}{2} \Rightarrow \underline{\alpha = 40{,}893^\circ} \rightarrow (*)$$

$$\boxed{d = \frac{4}{\sin 40{,}893^\circ} \approx 6{,}11 \text{ km}}$$



$c_1 = 2k$
$c_2 = 5k$

⑥ (zadaci - 5)

prav. trokut

$c_1 : c_2 = 2:5$

$|BC_1| = c_1$
$|AC_1| = c_2$

$$\frac{c_1}{c_2} = \frac{2}{5}$$

$$\alpha + \beta = 90^\circ \Rightarrow \beta = 90^\circ - \alpha$$

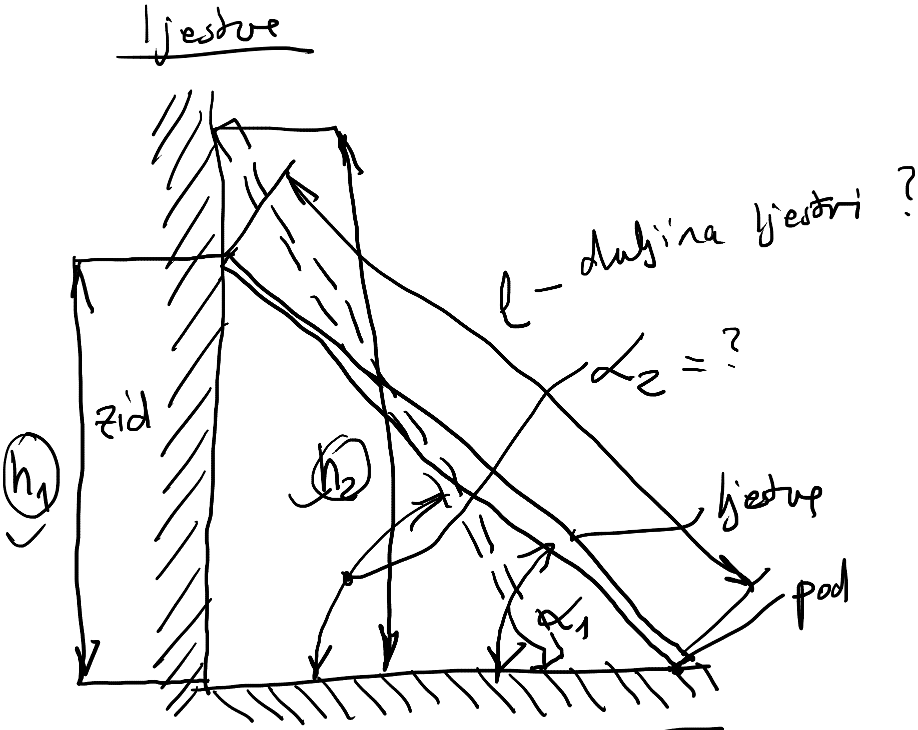
Euler-ov poučak: $v = \sqrt{5k \cdot 2k} = k\sqrt{10}$

$$\mathrm{tg}\,\alpha = \frac{v}{c_2} = \frac{\cancel{k}\sqrt{10}}{5\cancel{k}} = \frac{\sqrt{10}}{5} \Rightarrow \boxed{\alpha = 32{,}312^\circ = 32^\circ 18'}$$

$$\boxed{\beta = 90^\circ - 32{,}312^\circ = 57{,}688^\circ}$$

(7) (ispit 2 − 4)

$$\left.\begin{array}{l}\alpha_1 = 60^\circ \longrightarrow h_1 = 6{,}5\ m \\ \qquad\qquad\quad h_2 = 7\ m \longrightarrow \alpha_2 = ?\end{array}\right\}\ \Delta\alpha = \alpha_2 - \alpha_1 = ?$$



$$\frac{h_1}{l} = \sin\alpha_1$$

$$\Rightarrow l = \frac{h_1}{\sin\alpha_1} = \frac{6{,}5}{\sin 60^\circ}$$

$$l = \frac{13}{3}\sqrt{3}\ m$$

$$\sin\alpha_2 = \frac{h_2}{l} = \frac{7}{\frac{13}{3}\sqrt{3}}$$

$$\Rightarrow \alpha_2 = 68{,}851^\circ$$

$$\Delta\alpha = 68{,}851^\circ - 60^\circ = \boxed{8{,}851^\circ}$$

14. POUČCI O TROKUTU I PRIMJENE TRIGONOMETRIJE

① (Pr 4) $\alpha = ?$

$$\frac{(b+c)^2 - a^2}{bc} = 1 \quad / \cdot bc$$

$$(b+c)^2 - a^2 = bc$$

$$-a^2 = bc - (b+c)^2 \quad / \cdot (-1)$$

$$a^2 = (b+c)^2 - bc = b^2 + \underbrace{2bc - bc}_{bc} + c^2$$

$$a^2 = b^2 + bc + c^2 \quad (1)$$

Kosinusov poučak:

$$a^2 = b^2 + c^2 - 2bc\cos\alpha \quad (2) \quad / \cdot (-1)$$

$$-a^2 = -b^2 - c^2 + 2bc\cos\alpha$$

(1) + (2):

$$bc + 2bc\cos\alpha = 0 \quad / : bc$$

$$\cos\alpha = -\frac{1}{2} \Rightarrow \boxed{\alpha = 120^\circ = \frac{2}{3}\pi}$$

② (Pr 3)

$O_\Delta = 55$ cm

$\alpha = 46^\circ 27' = 46{,}45^\circ$

$\beta = 63^\circ 15' = 63{,}25^\circ$

$P_\Delta = ?$

$$P_\Delta = \frac{c^2 \sin\alpha \sin\beta}{2\sin\gamma}$$

$$\alpha + \beta + \gamma = 180^\circ$$

$$\Rightarrow \gamma = 180^\circ - (\alpha + \beta) =$$

$$\gamma = 180^\circ - (46{,}45^\circ + 63{,}25^\circ)$$

$$\gamma = 70{,}3^\circ$$

$c = ?$

$$O_\Delta = a + b + c = 55$$

$$a\sin\gamma = c\sin\alpha$$

$$b\sin\gamma = c\sin\beta$$

$$c\left(\frac{\sin\alpha}{\sin\gamma} + \frac{\sin\beta}{\sin\gamma} + 1\right) = 55$$

$$c\left(\frac{\sin 46{,}45^\circ}{\sin 70{,}3^\circ} + \frac{\sin 63{,}25^\circ}{\sin 70{,}3^\circ} + 1\right) = 55$$

$$\Rightarrow \quad c = 20{,}23 \text{ cm}$$

$$\boxed{P_\Delta = \frac{20{,}23^2 \cdot \sin 46{,}45^\circ \cdot \sin 63{,}25^\circ}{2\sin 70{,}3^\circ} = 140{,}7 \text{ cm}^2}$$

3 (Pr 5)

$$\underbrace{\overset{\alpha}{n-1},\ n,\ \overset{\gamma}{n+1}}_{\text{duljine stranica trokuta}} \quad - \text{ 3 uzastopna cijela broja}$$

$$\gamma = 2\alpha$$

$$O_\Delta = a + b + c$$

$$O_\Delta = n - 1 + n + n + 1 = 3n \quad ?$$

$$O_\Delta = ?$$

Poučak o sinusima:

$$\frac{a}{\sin\alpha} = \frac{c}{\sin\gamma}$$

$$\frac{n-1}{\sin\alpha} = \frac{n+1}{\sin 2\alpha}$$

$$\frac{n+1}{n-1} = \frac{\sin 2\alpha}{\sin\alpha} = \frac{2\cancel{\sin\alpha}\cos\alpha}{\cancel{\sin\alpha}} = 2\cos\alpha$$

$$\cos\alpha = \frac{1}{2}\,\frac{n+1}{n-1}$$

Poučak o kosinusima:

$$a^2 = b^2 + c^2 - 2bc\cos\alpha$$

$$(n-1)^2 = n^2 + (n+1)^2 - 2n(n+1)\cos\alpha$$

$$\vdots$$

$$-n^2 + 5n = 0 \quad /:n$$

$$n = 5$$

$$a = 4 \text{ cm} \quad b = 5 \text{ cm} \quad c = 6 \text{ cm} \longrightarrow \boxed{O = 4 + 5 + 6 = 15 \text{ cm}}$$

(4) (Pr 7)

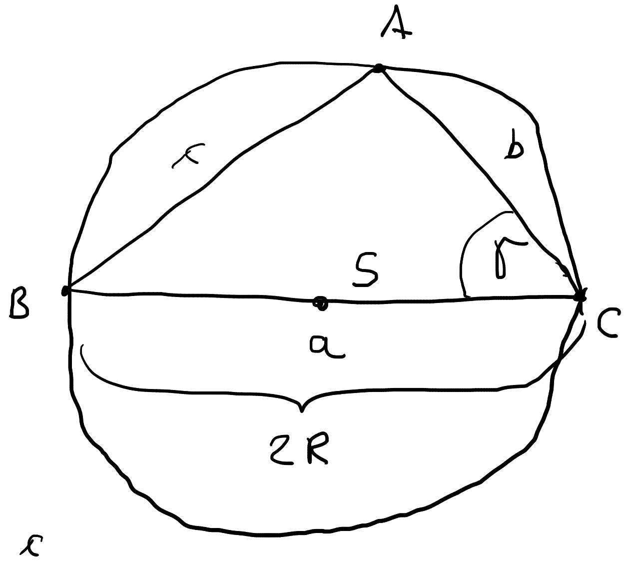
$\triangle ABC$

$a - b = 5\ cm \quad (1)$

$c = 7\ cm \quad (2)$

$R = \frac{7\sqrt{3}}{3}\ cm$

$a = ?$



poučak o sinusu:

$$\frac{c}{\sin\gamma} = 2R \implies \sin\gamma = \frac{c}{2R}$$

$$\sin\gamma = \frac{\cancel{7}}{2 \cdot \frac{\cancel{7}}{3}\sqrt{3}} = \frac{\sqrt{3}}{2} \implies \begin{array}{l} \gamma_1 = 60^\circ \quad (I) \\ \gamma_{II} = 120^\circ \quad (II) \end{array} \quad (3)$$

poučak o kosinusu:

$$c^2 = a^2 + b^2 - 2ab\cos\gamma \quad (4)$$

I) $\gamma = 60^\circ$ $(1), (2), (3) \rightarrow (4)$

$$7^2 = a^2 + (a-5)^2 - 2a(a-5)\underbrace{\cos 60^\circ}_{=\frac{1}{2}}$$

$$49 = a^2 + a^2 - 10a + 25 + (-2a^2 + 10a)\frac{1}{2}$$

$\vdots$

$$a^2 - 5a - 24 = 0 \rightarrow \begin{array}{l} \cancel{a_1 = -3\ cm} \\ a_2 = 8\ cm \end{array}$$

II) $\gamma = 120^\circ$

$$7^2 = a^2 + (a-5)^2 - 2a(a-5)\underbrace{\cos 120^\circ}_{=-\frac{1}{2}}$$

$\vdots$

$$3a^2 - 15a - 24 = 0 \quad /:3$$

$$a^2 - 5a - 8 = 0 \rightarrow \begin{array}{l} \cancel{a_3 \approx -1{,}3\ cm} \\ a_4 \approx 6{,}3\ cm \end{array}$$

za $\gamma = 60^\circ \rightarrow a = 8\ cm$

za $\gamma = 120^\circ \rightarrow a \approx 6{,}3\ cm$

5 (Pr 9)

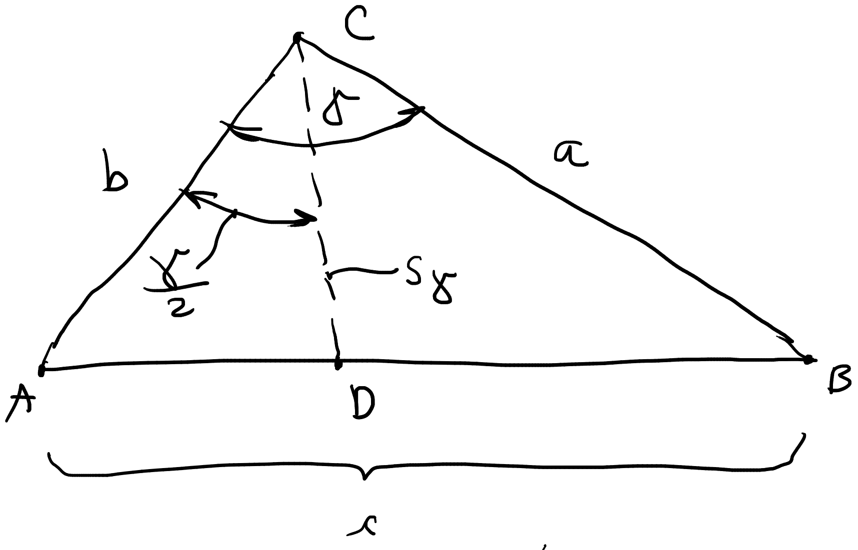
ΔABC

$\gamma = 120^\circ$

$ab = a + b$

$s_\gamma = ?$

$\hookrightarrow$ simetrala kuta γ



$$P_{\Delta ABC} = \frac{1}{2} ab \sin\gamma$$

$$P_{\Delta ABC} = \underbrace{P_{\Delta ADC}}_{= \frac{1}{2} s_\gamma b \sin\left(\frac{\gamma}{2}\right)} + P_{DBC} \qquad P_{DBC} = \frac{1}{2} s_\gamma a \sin\left(\frac{\gamma}{2}\right) \quad / \cdot 2$$

$$\underbrace{ab\sin 120^\circ}_{\frac{\sqrt{3}}{2}} = s_\gamma b \underbrace{\sin 60^\circ}_{\frac{\sqrt{3}}{2}} + s_\gamma a \underbrace{\sin 60^\circ}_{\frac{\sqrt{3}}{2}} \quad / \cdot \frac{2}{\sqrt{3}}$$

$$\left.\begin{array}{l} ab = s_\gamma b + s_\gamma a \quad (1) \\ ab = a + b \quad (2) \quad / \cdot (-1) \end{array}\right\} (+)$$

$$s_\gamma(a+b) = a + b \implies \boxed{s_\gamma = 1 \text{ cm}}$$

6 (zadaci - 4)

$a : b : c = 2 : 5 : 6$

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najveći kut γ

$a = 2k$

$b = 5k$

$c = 6k$

$$c^2 = a^2 + b^2 - 2ab\cos\gamma$$

$$(6k)^2 = (2k)^2 + (5k)^2 - 2 \cdot 2k \cdot 5k \cdot \cos\gamma$$

$$36k^2 = 4k^2 + 25k^2 - 20k^2\cos\gamma \quad / : k^2$$

$$36 = 29 - 20\cos\gamma$$

$$20\cos\gamma = -7$$

$$\Rightarrow \cos\gamma = -\frac{7}{20}$$

$$\boxed{\gamma = \arccos\left(-\frac{7}{20}\right) = 110{,}487^\circ = 110^\circ 29' 13''}$$

⑦ (zadatak 12)

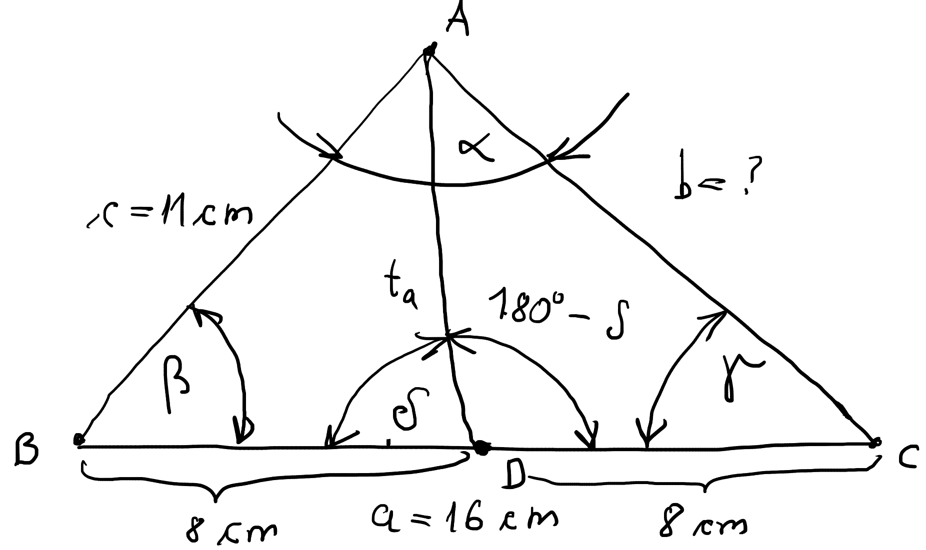
$\triangle ABC$

$t_a = 15$ cm − težišnica iz vrha A

$a = 16$ cm

$c = 11$ cm

$b = ?$



$$c^2 = \frac{a^2}{4} + t_a^2 - 2\cdot\frac{a}{2}\cdot t_a\cdot\cos\delta$$

$$11^2 = 16^2 + 15^2 - 2\cdot 16\cdot 15\cdot\cos\delta$$

$$\Rightarrow \cos\delta = 0{,}7$$

$$\delta = 45{,}573^\circ$$

$$180^\circ - \delta = 180^\circ - 45{,}573^\circ = 134{,}43^\circ$$

$$b^2 = t_a^2 + \left(\frac{a}{2}\right)^2 - 2\cdot t_a\cdot\frac{a}{2}\cdot\cos(180^\circ - \delta) \quad / \sqrt{\ }$$

$$\boxed{b = \sqrt{15^2 + 8^2 - 2\cdot 15\cdot 8\cdot\cos 134{,}43^\circ} = 19{,}3 \text{ cm}}$$

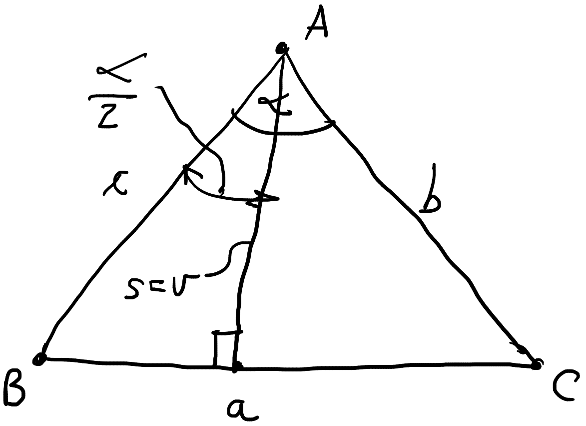
⑧ (zadatak 14)

jednakokračan trokut

$b = c = 15$ cm

$\alpha = 30^\circ$

$v = s = ?$



$$\frac{v}{c} = \cos\left(\frac{\alpha}{2}\right) = \cos 15^\circ$$

$$\boxed{v = c\cdot\cos 15^\circ = 15\cdot\cos 15^\circ = 14{,}5 \text{ cm}}$$

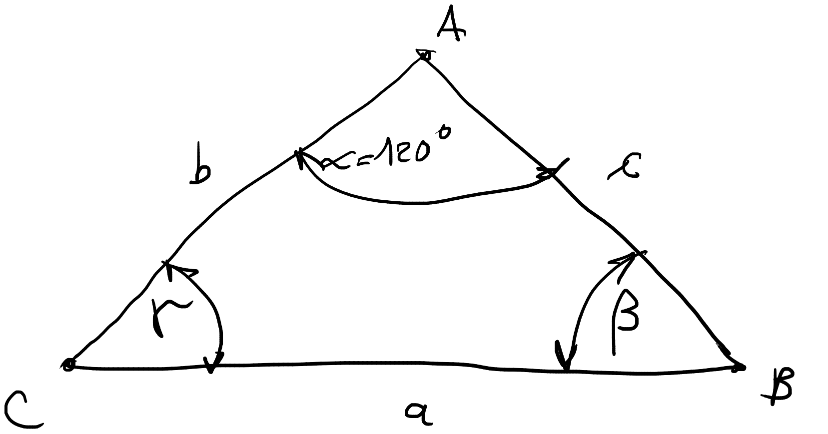
9 (ispit 1 – 6)

ΔABC

$a - b = b - c = 5\ cm$

$\alpha = 120°$

$a = ?$



$a^2 = b^2 + c^2 - 2bc\cos\alpha$

$a - b = 5 \Rightarrow a = 5 + b$

$b - c = 5 \Rightarrow c = b - 5$ $\Longrightarrow$ $c = 12,5 - 5 = 7,5\ cm$

$(b+5)^2 = b^2 + (b-5)^2 + b^2 - 5b$

$\cancel{b_1 = 0\ cm}$ $\ominus$

$b_2 = \frac{25}{2} = 12,5\ cm$

$a = \sqrt{b^2 + c^2 - 2bc\cos\alpha}$

$a = \sqrt{12,5^2 + 7,5^2 - 2 \cdot 12,5 \cdot 7,5 \cos 120°}$

$\boxed{a = 17,5\ cm}$

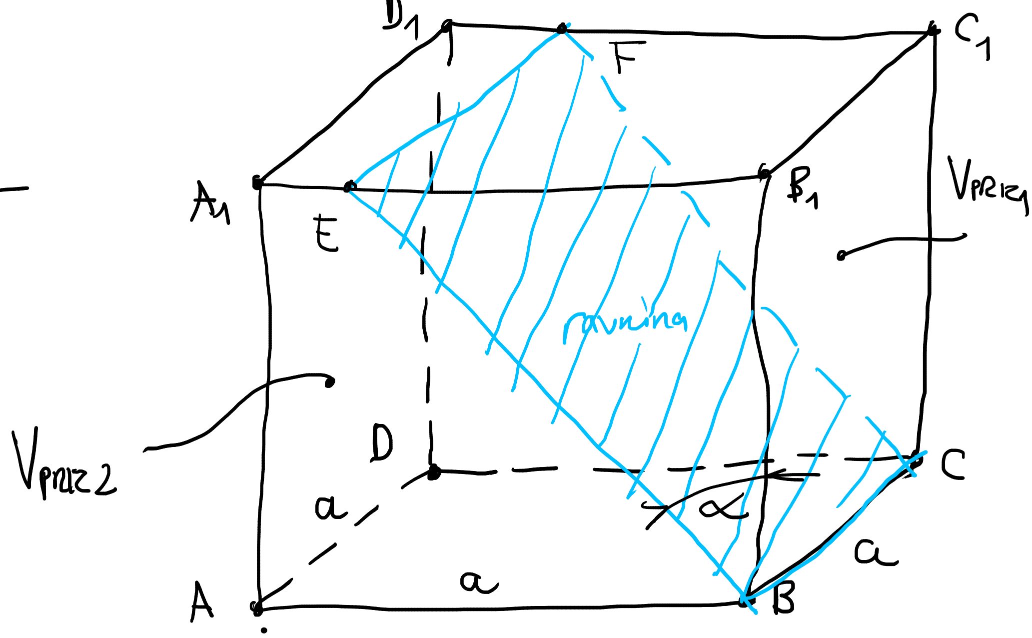
15. POLIEDRI I ROTACIJSKA TIJELA

① (Pr 3)

kocka

30°

$$\frac{V_{PRIZ2}}{V_{PRIZ1}} = ?$$



$$V_{PRIZ2} = V_K - V_{PRIZ1}$$

$$V_K = a^3$$

$$V_{PRIZ1} = P_{\Delta BB_1E} \cdot a$$

$$\underbrace{P_{\Delta BB_1E}}_{\text{pravokutni trokut}} = \frac{1}{2}a \cdot |\overline{EB_1}| \; ?$$

$$\frac{|\overline{EB_1}|}{a} = \operatorname{tg}\alpha = \operatorname{tg}30^\circ = \frac{\sqrt{3}}{3}$$

$$\Rightarrow |\overline{EB_1}| = a \cdot \frac{\sqrt{3}}{3}$$

$$P_{\Delta BB_1E} = \frac{1}{2}a \cdot a \cdot \frac{\sqrt{3}}{3} = \frac{\sqrt{3}}{6}a^2$$

$$V_{PRIZ1} = \frac{\sqrt{3}}{6}a^2 \cdot a = \frac{\sqrt{3}}{6}a^3$$

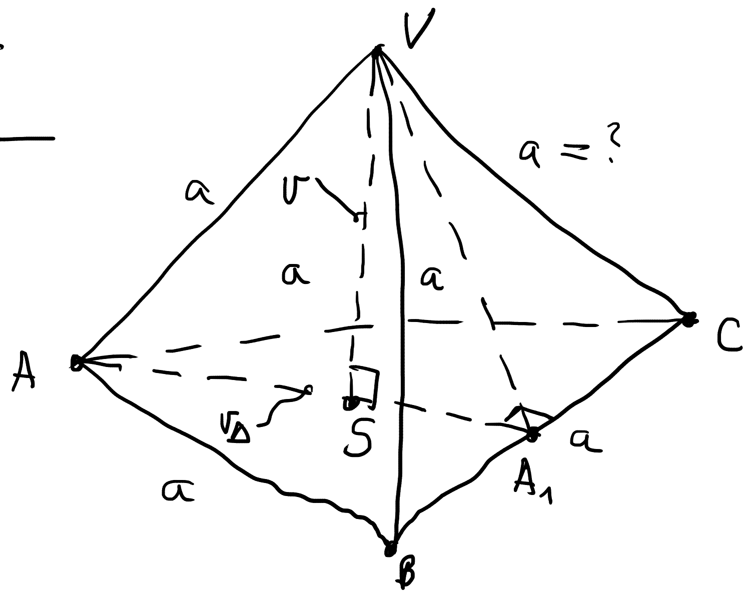
$$V_{PRIZ2} = a^3 - \frac{\sqrt{3}}{6}a^3 = \frac{6-\sqrt{3}}{6}a^3$$

$$\boxed{\frac{V_{PRIZ2}}{V_{PRIZ1}}} = \frac{\frac{6-\sqrt{3}}{\cancel{6}\,1}\cancel{a^3}}{\frac{\sqrt{3}}{\cancel{6}\,1}\cancel{a^3}} = \frac{6-\sqrt{3}}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{6\sqrt{3}-3}{3} = \boxed{2\sqrt{3}-1}$$

② pravilni tetraedar

$v = \sqrt{3}\ m$

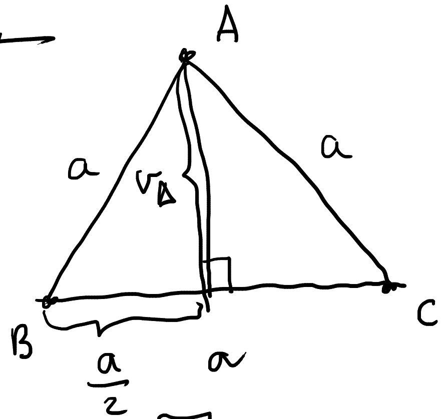
$V = ?$



$V = \frac{1}{3} B \cdot v$

$B = P_{\Delta ABC} = \frac{1}{2} a \cdot v_\Delta$

trokut ABC:



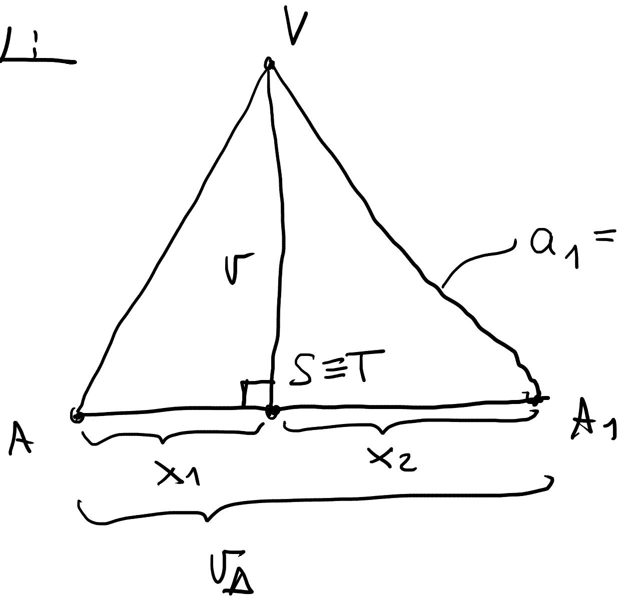
$v_\Delta = \sqrt{a^2 - \left(\frac{a}{2}\right)^2}$

$v_\Delta = \sqrt{\frac{4-1}{4} a^2} = \frac{\sqrt{3}}{2} a$

$B = \frac{1}{2} a \cdot \frac{\sqrt{3}}{2} a = \frac{\sqrt{3}}{4} a^2$

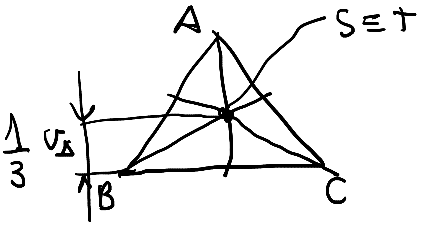
$V = \frac{1}{3} \cdot \frac{\sqrt{3}}{4} a^2 \cdot \sqrt{3} = \frac{3}{12} a^2 = \frac{1}{4} a^2$ $a = ?$

trokut ΔAA_1V:



$a_1 = v_\Delta = \frac{\sqrt{3}}{2} a$

trokut ABC



$x_1 = \frac{2}{3} v_\Delta = \frac{2}{3} \cdot \frac{\sqrt{3}}{2} a = \frac{\sqrt{3}}{3} a$

ΔSA_1V

$\left(\frac{\sqrt{3}}{2} a\right)^2 = \left(\frac{\sqrt{3}}{6} a\right)^2 + \left(\sqrt{3}\right)^2$

$\frac{3}{4} a^2 - \frac{3}{36} a^2 = 3 \Rightarrow a = \frac{3\sqrt{2}}{2}\ m$

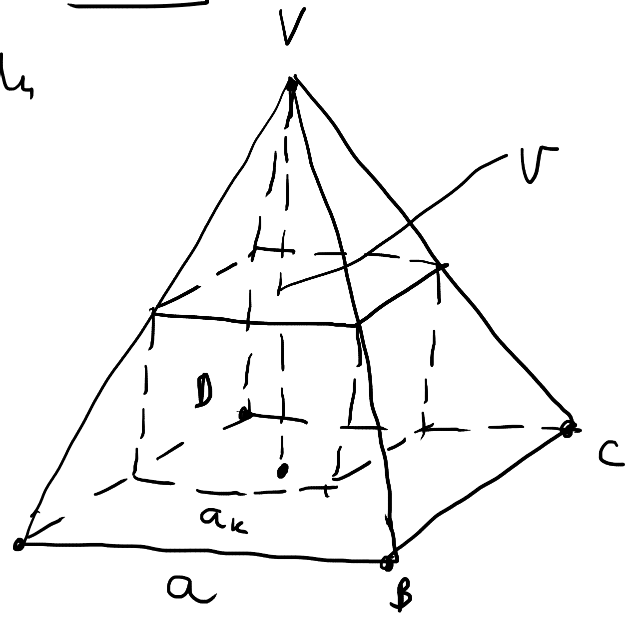
$$\boxed{V = \frac{1}{4}\cdot\left(\frac{3\sqrt{2}}{2}\right)^2 = \frac{1}{4}\cdot\frac{9}{2} = \frac{9}{8}\ m^3}$$

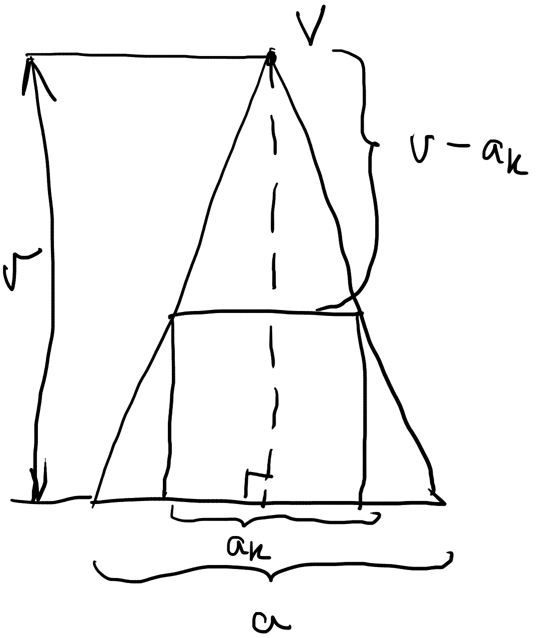
③ pravilna četverostrana piramida

$v = h = 2\ m$

$a = 1\ m$

$a_k = ?$





sličnost trokuta

$$\frac{a}{v} = \frac{a_k}{v - a_k}$$

$$\frac{1}{2} = \frac{a_k}{2 - a_k}$$

$$2a_k = 2 - a_k$$

$$3a_k = 2 \Rightarrow \boxed{a_k = \frac{2}{3}\ m}$$

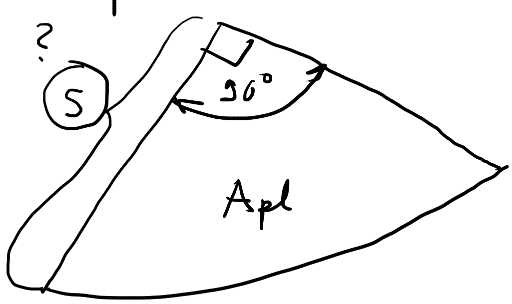
④ (Pr 7)

$A_{pl} = 100\pi\ cm^2$

$\alpha = 90°$

$v = ?$

plašt stošca



$$v = \sqrt{s^2 - r^2}$$

$$P_{kl} = \frac{s^2\pi\alpha}{360°} = A_{pl} = 100\pi$$

$$\frac{s^2\cancel{\pi}\cdot\cancel{90°}}{\cancel{360°}_4} = 100\cancel{\pi}$$

$s^2 = 400 \Rightarrow s = 20\ cm$

$$2r\pi = \frac{s\pi\alpha}{180^\circ} \Rightarrow \underline{r = 5 \text{ cm}}$$

$$\boxed{v = \sqrt{20^2 - 5^2} = \sqrt{375} = 5\sqrt{15}\text{ cm}}$$

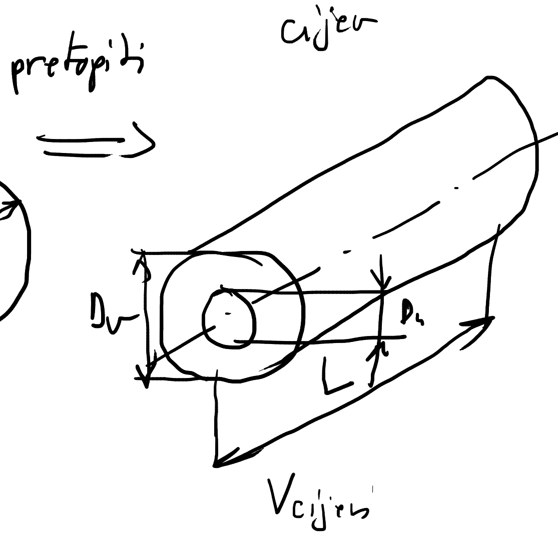
5 (Pr 8)

$R = 15$ cm → kugla

$D = D_v = 42$ mm

$d = D_u = 40$ mm

$L = ?$



$V_k = V_{cijevi}$

$$\underline{V_k} = \frac{4}{3}R^3\pi = \frac{4}{3}\cdot 7{,}5^3\pi = \underline{562{,}5\,\pi\text{ cm}^3}$$

$$V_{cijevi} = \left(\frac{D_v^2 - D_u^2}{4}\right)\pi L = \left(\frac{4{,}2^2 - 4^2}{4}\right)\pi L \approx 0{,}41\,\pi L \ [\text{cm}^3]$$

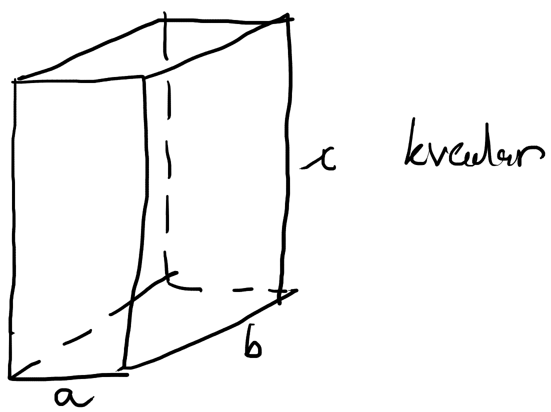
$$562{,}5\,\pi = 0{,}41\cdot\pi L \Rightarrow \boxed{L = 1371{,}951\text{ cm} = 13{,}72\text{ m}}$$

6 (ispit 1-1)

$a : b : c = 1 : 2 : 3$

$V = 162\text{ cm}^3$

$O = ?$



$$O = 2(ab + bc + ac)$$

$$V = abc = 162$$

$$\frac{a}{b} = \frac{1}{2} \Rightarrow b = 2a$$

$$\frac{a}{c} = \frac{1}{3} \Rightarrow c = 3a$$

$$a\cdot 2a\cdot 3a = 162$$

$$6a^3 = 162 \quad | :6$$

$$a^3 = 27 \Rightarrow a = 3\ cm$$

$$b = 2 \cdot 3 = 6\ cm$$

$$c = 3 \cdot 3 = 9\ cm$$

$$\boxed{O = 2(3 \cdot 6 + 6 \cdot 9 + 3 \cdot 9) = 198\ cm^2}$$