1. REALNI BROJEVI

① (Pr 4)

1)

$$(n-\cancel{2})+(n-\cancel{1})+n+(n+\cancel{1})+(n+\cancel{2})=5n$$ djeljiv je sa 5

2)

$$(2n-\cancel{2})+2n+(2n+\cancel{2})=6n$$ – djeljiv je sa 6

3)

$$(2n-\cancel{3})+(2n-\cancel{1})+(2n+\cancel{1})+(2n+\cancel{3})=8n$$ – djeljiv sa 8

② (Pr 5)

$$Z_n=561$$
$$n=?$$

$$Z_n=n\cdot\frac{(n+1)}{2}$$

$$n\cdot\frac{(n+1)}{2}=561 \quad /\cdot 2$$

⋮

$$n^2+n=1122$$

$$n^2+n-1122=0$$

$$\cancel{n_1=-34}$$ nije negativno rješenje!

$$\boxed{n_2=33}$$

③ (Pr 8)

$$x+y=750 \Rightarrow x=750-y$$
$$0{,}08\cdot x+0{,}24\cdot y=0{,}112\cdot 750$$

$$0{,}08\cdot(750-y)+0{,}24\cdot y=84$$

⋮

$$0{,}16\,y=24$$

$$\boxed{y=150} \Rightarrow \boxed{x=600}$$

4) (fr 9)

a, b

$$\frac{(a-b):(a+b):ab = 1:2:6}{\frac{a}{b} = ?}$$

$$\frac{a-b}{a+b} = \frac{1}{2} \Rightarrow a = 3b$$

$$\frac{a+b}{ab} = \frac{2}{6} = \frac{1}{3} \qquad \frac{3b+b}{3b \cdot b} = \frac{1}{3}$$

$$\frac{4\cancel{b}^{1}}{3b^{\cancel{2}}{}_{1}} = \frac{1}{3} \Rightarrow \boxed{b=4}$$

$$\boxed{a=12}$$

5)

$$\frac{12_{(n)} + 23_{(n)} = 40_{(n)}}{n = ?}$$

$$12_{(n)} = 1 \cdot n^1 + 2 \cdot n^0 = n+2$$

$$23_{(n)} = 2 \cdot n^1 + 3 \cdot n^0 = 2n+3$$

$$40_{(n)} = 4 \cdot n^1 + 0 \cdot n^0 = 4n$$

$$n+2+2n+3 = 4n$$

$$-n = -5$$

$$\boxed{n=5}$$

6)

$$\frac{x+y}{2} = 5 \quad / \cdot 2$$

$$\sqrt{x \cdot y} = 4 \quad /^2$$

$$x+y = 10 \Rightarrow y = 10-x$$

$$xy = 16$$

$x \cdot (10 - x) = 16$

$-x^2 + 10x - 16 = 0$

$\vdots$

$x_1 = 2 \Rightarrow y_1 = 8$

$x_2 = 8 \Rightarrow y_2 = 2$

$|x_1 - y_1| = ?$

$|x_2 - y_2| = ?$

$|2-8| = |8-2| = \boxed{6}$

7

$\sqrt[4]{\sqrt[3]{x^2}} : \sqrt[3]{x^2 \cdot \sqrt{x}} =$

$= \sqrt[4]{x^{\frac{2}{3}}} : \sqrt[3]{x^2 \cdot x^{\frac{1}{2}}} = \left(x^{\frac{\cancel{2}^{1}}{3}}\right)^{\frac{1}{\cancel{4}2}} : \underbrace{\sqrt[3]{x^{\frac{5}{2}}}}_{\left(x^{\frac{5}{2}}\right)^{\frac{1}{3}}}$

$= x^{\frac{1}{6}} : x^{\frac{5}{6}} = \dfrac{x^{\frac{1}{6}}}{x^{\frac{5}{6}}} = x^{\frac{1}{6} - \frac{5}{6}} = x^{\frac{-4}{6}} = x^{-\frac{2}{3}}$

$= \boxed{\sqrt[3]{x^{-2}}}$

2. ALGEBARSKI IZRAZI

① (fr 3)

$$x+y=1$$
$$xy=2$$

$$x^2+y^2=?$$
$$x^3+y^3=?$$

$$(x+y)^2=x^2+2xy+y^2=1$$

$$\boxed{x^2+y^2}=1-2xy=1-2\cdot 2=\boxed{-3}$$

$$\boxed{(x+y)^3}=x^3+3x^2y+3xy^2+y^3=1$$

$$\boxed{x^3+y^3}=1-3x^2y-3xy^2=1-3\underbrace{xy}_{=2}\underbrace{(x+y)}_{=1}$$

$$=1-3\cdot 2\cdot 1=\boxed{-5}$$

②

$$\left(1-\frac{1}{1-\frac{a^2}{a-1}}\right)\cdot\frac{a^3+1}{a^2}$$

$$1-\frac{a^2}{a-1}=\frac{a-1-a^2}{a-1}=\frac{-a^2+a-1}{a-1}$$

$$1+\frac{a-1}{a^2-a+1}=\frac{a^2-a+1+a-1}{a^2-a+1}=\frac{a^2}{a^2-a+1}$$

$$\frac{a^3+1}{a^2}=\frac{(a+1)(a^2-a+1)}{a^2}$$

$$\frac{\cancel{a^2}^{\,1}}{\cancel{a^2-a+1}_{\,1}}\cdot\frac{(a+1)\cancel{(a^2-a+1)}^{\,1}}{\cancel{a^2}_{\,1}}=\boxed{a+1}$$

③ (Pr 7)

$$\left(\frac{a+b}{a^2b-ab^2}-\frac{a-b}{a^2b+ab^2}\right)\cdot\frac{a^4-b^4}{4}$$

$$\frac{a+b}{a^2b-ab^2}-\frac{a-b}{a^2b+ab^2}=\frac{a+b}{ab(a-b)}-\frac{a-b}{ab(a+b)}$$

$$=\frac{(a+b)^2-(a-b)^2}{ab(a-b)(a+b)}$$

$(a-b)(a+b)$

$$\frac{a^4-b^4}{4}=\frac{(a^2-b^2)(a^2+b^2)}{4}$$

$$\frac{(a+b)^2-(a-b)^2}{ab\cancel{(a-b)}\cancel{(a+b)}}\cdot\frac{\cancel{(a-b)(a+b)}(a^2+b^2)}{4}$$

$$=a^2+2ab+b^2-(a^2-2ab+b^2)=4ab$$

$$=\frac{\cancel{4ab}}{\cancel{ab}}\cdot\frac{a^2+b^2}{\cancel{4}}=\boxed{a^2+b^2}$$

④

$$(a+b\sqrt{2})^2=9+4\sqrt{2}$$

$ab=?$

$$a^2+2ab\sqrt{2}+2b^2=9+4\sqrt{2}$$

$$a^2+2b^2=9$$

$$2ab\sqrt{2}=4\sqrt{2}\Rightarrow\boxed{ab=2}$$

(5)

$$\frac{2^m \cdot 3^{n-1} - 2^{m-1} \cdot 3^n}{2^m \cdot 3^n}$$

$$= \frac{\cancel{2^m} \cdot 3^{n-1}}{\cancel{2^m} \cdot 3^n} - \frac{2^{m-1} \cdot \cancel{3^n}}{2^m \cdot \cancel{3^n}} = 3^{\cancel{n}-1-\cancel{n}} - 2^{\cancel{m}-1-\cancel{m}}$$

$$= \frac{1}{3} - \frac{1}{2} = \frac{2-3}{6} = \boxed{-\frac{1}{6}}$$

(6) $8x^3 - 36x^2 + ax + b$ – kub binoma od $mx+n$

$m+n=?$

$$(mx+n)^3 = m^3x^3 + 3m^2x^2n + 3mxn^2 + n^3$$

$$= 8x^3 - 36x^2 + ax + b$$

$m^3 = 8 \Rightarrow m=2$

$3m^2n = -36 \Rightarrow \quad 3 \cdot 2^2 \cdot n = -36$

$3mn^2 = a \quad\quad n = -3$

$n^3 = b$

$$\boxed{m+n = 2-3 = -1}$$

3. POLINOMI I ALGEBARSKE JEDNADŽBE

① (Pr 3)

$$\frac{f(x+1) = x^3 - 3x+1}{f(-2) = ?}$$

$$x+1 = -2$$

$$x = -2-1 = -3$$

$$f(-2) = f(-3+1) = (-3)^3 - 3\cdot(-3)+1 = \boxed{-17}$$

② (Pr 5)

$$\frac{1}{x^3+1} = \frac{a}{x+1} + \frac{bx+c}{x^2-x+1} \quad / \cdot (x^3+1)$$

$$1 = a \underbrace{\frac{x^3+1}{x+1}}_{x^2-x+1} + (bx+c)\cdot \underbrace{\frac{x^3+1}{x^2-x+1}}_{x+1}$$

$$1 = a(x^2-x+1) + (bx+1)(x+1)$$

$$1 = (a+b)x^2 + (-a+b+c)x + a+c$$

teorem o jednakosti polinoma: $P = Q$

$$a+c = 1 \Rightarrow c = 1-a$$

$$a+b = 0 \Longrightarrow a = -b$$

$$\frac{-a+b+c = 0}{b+b+\underbrace{(1-a)}_{1+b} = 0}$$

$3b = -1 \Rightarrow \boxed{b = -\frac{1}{3} \quad a = \frac{1}{3} \quad x = 1 - \frac{1}{3} = \frac{2}{3}}$

(3) (Pr 8)

$P(x) = 6x^3 - 7x^2 - 16x + a$?

da bude djeljiv sa $x-2$

↳ ostatak $R(x) = 0$

$$(\cancel{6x^3} - 7x^2 - 16x + a) : (x-2) = 6x^2 + 5x - 6$$
$$-(\cancel{6x^3} - 12x^2)$$
$$\cancel{5x^2} - 16x$$
$$-(\cancel{5x^2} - 10x)$$
$$\cancel{-6x} + a$$
$$-(\cancel{-6x} + 12)$$
$$R(x) = a - 12 = 0 \Rightarrow \boxed{a = 12}$$

(4) $f(x) = ax^{11} - x^{10} + x^9 - x^8 + \dots + x - 1$

$a = ?$

$S = 13$

$a - 1 = 13 \Rightarrow \boxed{a = 14}$

5 (17 - zadaci)

$f(x) = x^2 + ax + b$

sa $x+1$ $\quad R_1(x) = 1$

sa $x-1$ $\quad R_2(x) = 3$

$a, b = ?$

$$(\cancel{x^2} + ax + b) : (x+1) = x + (a-1)$$
$$-(\cancel{x^2} + x)$$
$$\cancel{x(a-1)} + b$$
$$-(\cancel{x(a-1)} + (a-1))$$
$$b - (a-1) = 1 \Rightarrow a = b$$

$$(\cancel{x^2} + ax + b) : (x-1) = x + (a+1)$$
$$-(\cancel{x^2} - x)$$
$$\cancel{x(a+1)} + b$$
$$-(\cancel{x(a+1)} - (a+1))$$
$$b + (a+1) = 3 \Rightarrow a + b = 2$$

$$2a = 2 \Rightarrow \boxed{\begin{array}{l} a = 1 \\ b = 1 \end{array}}$$

⑥ (10 – ispit 1)

$$x^3 + 3x^2 + ax + b = 0$$

$x_{1,2} = -1$ – dvostruko rješenje

$x_3 = ?$

$$(x+y)^3 = x^3 + 3x^2y + 3xy^2 + y^3 \Longrightarrow y^3 = b$$

$$x^3 + 3x^2 + ax + b$$

$\Downarrow$ $\Downarrow$

$y = 1$ $3y = a$

$a = 3$

$b = 1$

$$x^3 + 3x^2 + 3x + 1 = 0$$

$$\boxed{x_3 = -1}$$

⑦

$$\frac{x^2}{x^3-1} = \frac{a}{x-1} + \frac{bx+c}{x^2+x+1} \quad / \cdot (x^3-1)$$

$a + b + c = ?$

$$x^2 = a(x^2+x+1) + (bx+c)(x-1)$$

$$x^2 = (a+b)x^2 + (a-b+c)x + (a-c)$$

$a + b = 1$

$a - b + c = 0 \Longrightarrow a - b + a = 0$

$2a - b = 0$

$a - c = 0 \Longrightarrow a = c$

$b = 2a$

$a + 2a = 1$

$3a = 1 \Longrightarrow a = \frac{1}{3}$ $b = \frac{2}{3}$ $c = \frac{1}{3}$

$$\boxed{a+b+c = \frac{1}{3} + \frac{2}{3} + \frac{1}{3} = \frac{4}{3}}$$