25. LIMES FUNKCIJA

①

a) $\lim_{x\to 0}\frac{\sqrt{x+1}-1}{x}$ f(x) g(x) $=\lim\frac{f'(x)}{g'(x)}$

↳ L' Hospitalovo pravilo

$$\frac{\left((x+1)^{\frac{1}{2}}-1\right)'}{(x)'}=\frac{\frac{1}{2}(x+1)^{-\frac{1}{2}}}{1}$$

$$\left(\frac{\sqrt{0+1}-1}{0}=\frac{0}{0}\right)$$

$$\lim_{x\to 0}\left[\frac{1}{2}(x+1)^{-\frac{1}{2}}\right]=\frac{1}{2}$$

↳ = 0

$\underbrace{\qquad}_{1}$

$x\to 0$

$$f(x)=\frac{\sqrt{x+1}-1}{x}$$

$x=0,1 \longrightarrow f(x=0,1)\approx 0,48809$

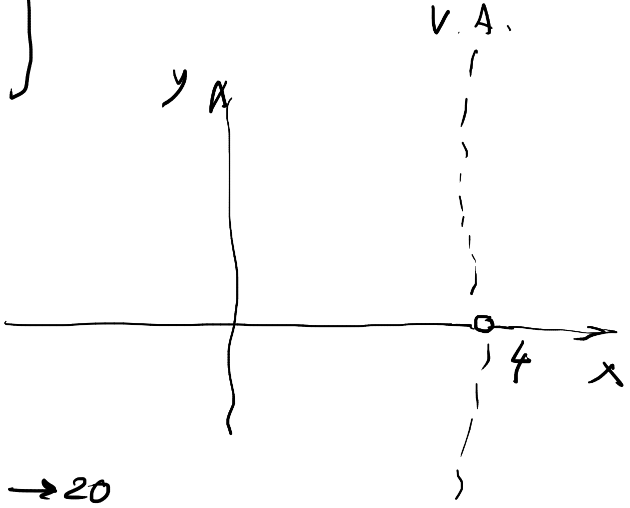
$x=0,01 \longrightarrow f(x=0,01)\approx 0,49876$

$x=0,001 \longrightarrow f(x=0,001)\approx 0,49999$

$x=0,0001 \longrightarrow$ ......

$\to 0,5$

b) $\lim_{x\to 4}\frac{x^{\frac{5}{2}}-32}{x-4}$ f(x)



$x=3,9 \to f(x=3,9)\approx 19,62657$

$x=3,99 \to f(x=3,99)\approx 19,96252$

$x=3,999 \to f(x=3,999)\approx 19,99625$

$x=3,9999 \to \approx 19,99963$

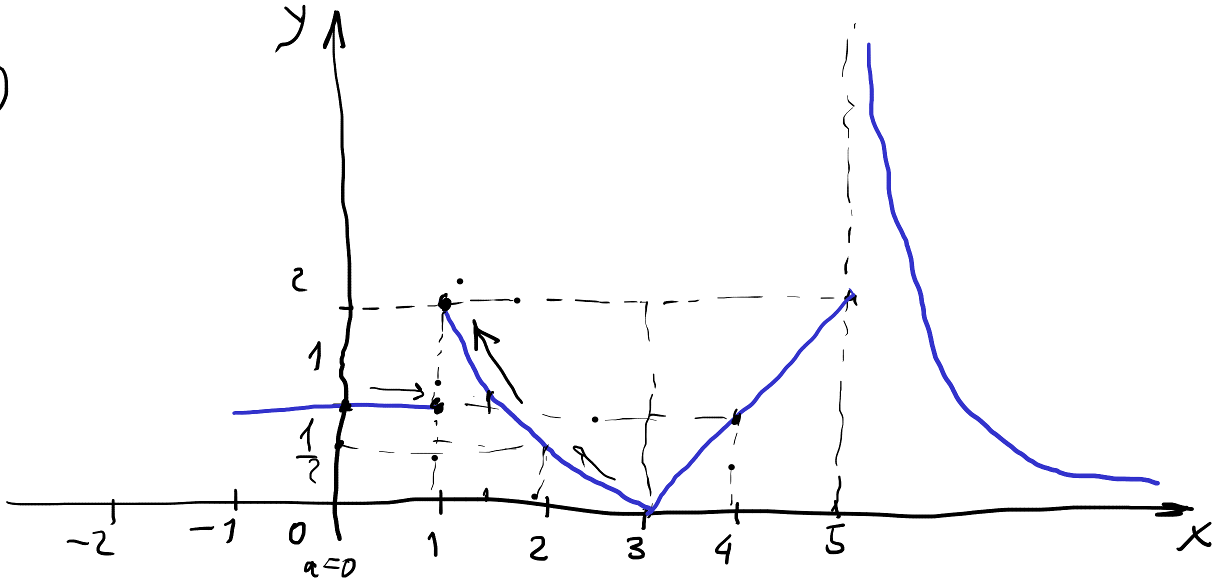
$\to 20$

Provjera: L' Hospitalovo pravilo:

$$\frac{f'(x)}{g'(x)}=\frac{\frac{5}{2}x^{\frac{3}{2}}}{1}=\frac{5}{2}x^{\frac{3}{2}}$$

$$\lim_{x\to 4}\left(\frac{5}{2}x^{\frac{3}{2}}\right)=\frac{5}{2}4^{\frac{3}{2}}=\frac{5}{2}\left(2^{\overset{1}{\cancel{2}}}\right)^{\frac{3}{\cancel{2}_1}}=\frac{5}{2}\cdot 8=\frac{40}{2}=20$$

② (zad 5)



a)

	$f(a)$	$\lim\limits_{x\to a} f(x)$	$\lim\limits_{x\to a^-} f(x)$	$\lim\limits_{x\to a^+} f(x)$
$a=0$	1	1	1	1
$a=1$	2	ne postoji	1	2
$a=2$	$\frac{1}{2}$	$\frac{25}{8}$	$\frac{1}{2}$	$\frac{1}{2}$
$a=3$	0	0	0	0
$a=4$	4	1	1	1
$a=5$	2	ne postoji	2	∞

$$f(x) = \begin{cases} c, & x < 1 \\ b(x-3)^2 & 1 \le x < 3 \\ \dfrac{x^2 + dx + 12}{x-a}, & 3 \le x \le 5,\quad x \ne 4 \\ e & x = 4 \\ \dfrac{1}{x-f}, & x > 5 \end{cases}$$

$c=1$

$b=\frac{1}{2}$

$d=7$

$a=4$

$e=2$

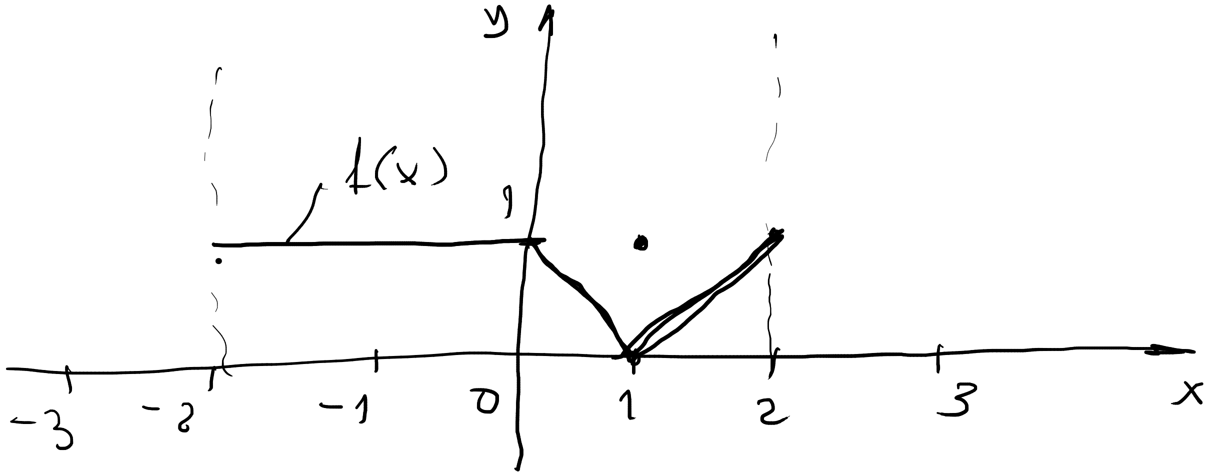
$f=5$

③

1) $D_f = [-2, 2]$

2) $f(-2) = f(-1) = f(1) = f(2) = 1$

3) $\lim_{x \to 1^-} f(x) = \lim_{x \to 1^+} f(x) = 0$



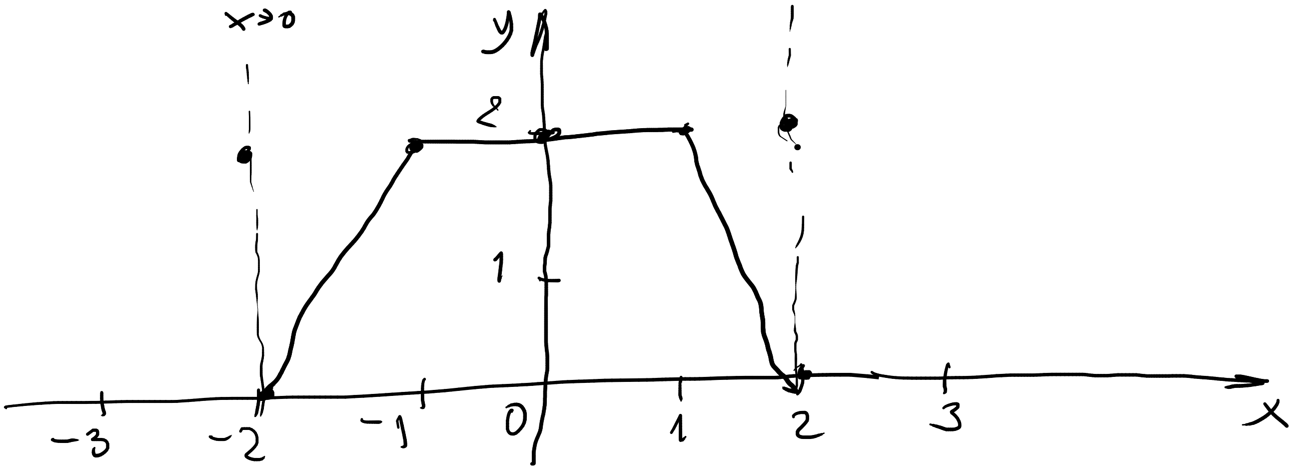
④

1) $D_f = [-2, 2]$

2) $f(-2) = f(0) = f(2) = 2$

3) $\lim_{x \to -2^-} f(x) = \lim_{x \to 2^-} f(x) = 0$

4) $\lim_{x \to 0} f(x) = 2$



⑤
(zad 11)

1) $$\lim_{x\to 0}\frac{x^2-1}{x+3} = \frac{0^2-1}{0+3} = -\frac{1}{3}$$

2) $$\lim_{a\to 2}\frac{a^2-2a+1}{a+1} = \frac{2^2-2\cdot 2+1}{2+1} = \frac{1}{3}$$

3) $$\lim_{x\to 3}\frac{2x+5}{2+x^2} = \frac{2\cdot 3+5}{2+3^2} = \frac{6+5}{2+9} = \frac{11}{11} = 1$$

⑥
(zad 13)

1) $$\lim_{x\to 2}\frac{x+2}{x^2-4} = \lim_{x\to 2}\frac{\overset{1}{\cancel{x+2}}}{(x-2)\underset{1}{\cancel{(x+2)}}} = \lim_{x\to 2}\frac{1}{x-2} = \frac{1}{0} -$$

ne postoji!

$$\lim_{x\to 2}\frac{1}{2x} = \frac{1}{2\cdot 2} = \frac{1}{4}$$

2) $$\lim_{x\to 2}\frac{x-2}{x^2-4} = \lim_{x\to 2}\frac{\cancel{x-2}}{\cancel{(x-2)}(x+2)} = \lim_{x\to 2}\frac{1}{x+2} = \frac{1}{4}$$ ✓

3) $$\lim_{u\to 1}\frac{\overset{1}{\cancel{u-1}}}{\boxed{u^2+2u-3}} = \lim_{u\to 1}\frac{1}{u+3} = \frac{1}{4}$$

$$L = \cancel{(u-1)}(u+3)$$

4) $$\lim_{n\to 1}\frac{n-1}{\underbrace{n^2-2n+1}_{(n-1)^2}} = \lim_{n\to 1}\frac{1}{n-1} = \frac{1}{0} -$$ ne postoji

$$\frac{1}{2n-2} \qquad \lim_{n\to 1}\left(\frac{1}{2n-2}\right) = \frac{1}{2\cdot 1-2} = \frac{1}{0} -$$

$$\frac{f''(x)}{g''(x)} = \frac{0}{2} = 0$$ ✓

⑦

(Zad 15)

1) $$\lim_{x\to 0} \underbrace{\boxed{\frac{\sqrt{x+4}-2}{x}}}_{f(x)} = \frac{2-2}{0} = \frac{0}{0}$$

$$\frac{\sqrt{x+4}-2}{x} \cdot \frac{\sqrt{x+4}+2}{\sqrt{x+4}+2} = \frac{x+4-4}{x(\sqrt{x+4}+2)} = \frac{\overset{1}{\cancel{x}}}{\underset{1}{\cancel{x}}(\sqrt{x+4}+2)}$$

$$\lim_{x\to 0}\left(\frac{1}{\sqrt{x+4}+2}\right) = \frac{1}{2+2} = \frac{1}{4}$$ ✓

2) $$\lim_{x\to 5} \frac{\overset{1}{\cancel{\sqrt{x}-\sqrt{5}}}}{\underbrace{x-5}} = \frac{0}{0}$$

$$\hookrightarrow = (\sqrt{x}+\sqrt{5})\underset{1}{\cancel{(\sqrt{x}-\sqrt{5})}}$$

$$\lim_{x\to 5} \frac{1}{\sqrt{x}+\sqrt{5}} = \frac{1}{\sqrt{5}+\sqrt{5}} = \frac{1}{2\sqrt{5}}$$

⑧

(Zad 16)

5) $$\lim_{x\to\infty} \frac{2+3x-x^2}{3x^2+2x+1} \begin{array}{l} /:x^2 \\ /:x^2 \end{array} = \lim_{x\to\infty} \frac{\overbrace{\frac{2}{x^2}+\frac{3}{x}}^{=0}-1}{3+\underbrace{\frac{2}{x}+\frac{1}{x^2}}_{=0}} = -\frac{1}{3}$$

6) $$\lim_{x\to\infty} \frac{x^4+2x}{1-x^2+3x^4} \begin{array}{l} /:x^4 \\ /:x^4 \end{array} = \lim_{x\to\infty} \frac{1+\boxed{\frac{2}{x^3}}=0}{\underbrace{\frac{1}{x^4}-\frac{1}{x^2}}_{=0}+3} = \frac{1}{3}$$

26. VJEROJATNOST

① (Zad 2)

dvije kocke - bacaju

Događaji $A = \{$zbroj brojeva je neparan$\}$

$B = \{$pojavio se broj 1$\}$

$C = \{$na obje kocke pao je broj 1$\}$

AB

AC

BC

$A \cup C$

$\bar{A} B$

$A = \{(m,n) : m+n \in \{3,5,7,9,11\}, m,n \in \{1,2,\dots 6\}\}$

$= \{12, 14, 16, 21, 23, 25, 32, 34, 36, 41, 43, 45, 52, 54, 61, 63, 65\}$

$B = \{(m,n) : m=1$ ili $n=1, m,n \in \{1,2,\dots 6\}\}$

$= \{11, 12, 13, 14, 15, 16, 21, 31, 41, 51, 61\}$

$C = \{11\}$

$AB = \{12, 21, 14, 41, 16, 61\}$

$AC = \emptyset$

$BC = \{11\}$

$A \cup C = \{11, 12, 14, 16, 21, 23, \dots 65\}$

$\bar{A} B = \{11, 13, 31, 15, 51\}$

② deset kartica

(Zad 18) $1, \dots, 10$

1) izvučemo odjednom obje kartice – da će se izvući jedan paran i neparan broj

→ broj povoljnih događaja

$$p = \binom{M}{N}$$

└ broj svih događaja

vjerojatnost

$$M = \binom{5}{1} \cdot \binom{5}{1} = 5 \cdot 5 = 25$$

$$N = \binom{10}{2} = 45$$

$$p = \binom{25}{45} = \frac{5}{9}$$

2) izvučemo prvu karticu, a nakon nje drugu;

$$p = \binom{M}{N} = \binom{25}{45} = \frac{5}{9}$$

└ = 45

$$M = \binom{5}{1} \cdot \binom{5}{1} = 5 \cdot 5 = 25$$

3) izvučemo prvu karticu, vratimo ju u snop i izvučemo drugu;

$$N = \binom{10}{1} \cdot \binom{10}{1} = 10 \cdot 10 = 100$$

$$p = \binom{\textcircled{M}}{N} = ?$$

$$M' = \binom{5}{1} \cdot \binom{5}{1} = 5 \cdot 5 = 25$$

$$M = 2 \cdot M' = 2 \cdot 25 = 50$$

$$\boxed{p = \binom{50}{100} = \frac{1}{2}}$$

③

(zad 20)

biramo na sreću neki prirodni broj između 1 i 30, mora biti djelitelj broja 30

$p = ?$

djelitelji broja 30 – $\underbrace{1, 2, 3, 5, 6, 10, 15, 30}_{n=8}$

$$\boxed{p = \frac{n}{30} = \frac{8}{30} = \frac{4}{15}}$$