7. KVADRATNA FUNKCIJA, JEDNADŽBE I NEJEDNADŽBE

① (Pr 1)

$$\frac{5}{4x^2-36}+\frac{1}{5x}=\frac{1}{4x+12}-\frac{1}{3x-x^2}$$

$$\frac{5}{\underbrace{4(x^2-9)}_{\underbrace{(x-3)(x+3)}_{\substack{x\neq -3\\ x\neq 3}}}}+\frac{1}{5x}=\frac{1}{4(x+3)}-\underbrace{\frac{1}{x(3-x)}}_{-\left(-\frac{1}{x(x-3)}\right)}\Bigg/\cdot 4(x-3)(x+3)5x$$

$\to x\neq 0$

$$5\cdot 5x+4(x-3)(x+3)=5x(x-3)+4\cdot 5\cdot(x+3)$$

$$25x+4(x^2-9)=5x^2-15x+20x+60$$

$\vdots$

$$x^2-20x+96=0$$

$$x_{1,2}=\frac{-b\pm\sqrt{b^2-4ac}}{2a}=\frac{20\pm\sqrt{(-20)^2-4\cdot 1\cdot 96}}{2\cdot 1}$$

$$\boxed{\begin{array}{l}x_1=8\\ x_2=12\end{array}}$$

② (Pr 2)

$$\frac{x+1}{2x^2}+\frac{3x^2}{x+1}=\frac{5}{2}$$

$$\frac{1}{2}\cdot\underbrace{\frac{x+1}{x^2}}_{=t}+3\underbrace{\frac{x^2}{x+1}}_{=\frac{1}{t}}=\frac{5}{2}$$

$$\frac{1}{2}t+3\cdot\frac{1}{t}=\frac{5}{2}\Big/\cdot 2t$$

$$t^2+6=5t$$

$$t^2 - 5t + 6 = 0$$

$$t_{1,2} = \frac{-(-5) \pm \sqrt{(-5)^2 - 4 \cdot 1 \cdot 6}}{2 \cdot 1} = \frac{5 \pm 1}{2}$$

$\underline{t_1 = 2}$ 1)

$\underline{t_2 = 3}$ 2)

1) $t_1 = 2$

$$\frac{x+1}{x^2} = 2 \quad / \cdot x^2$$

$$x + 1 = 2x^2$$

$$2x^2 - x - 1 = 0$$

$$x_{1,2} = \frac{-(-1) \pm \sqrt{(-1)^2 - 4 \cdot 2 \cdot (-1)}}{2 \cdot 2} = \frac{1 \pm 3}{4}$$

$$\boxed{\begin{array}{l} x_1 = -\frac{1}{2} \\ x_2 = 1 \end{array}}$$

2) $t_2 = 3$

$$\frac{x+1}{x^2} = 3 \implies 3x^2 - x - 1 = 0$$

$$x_{1,2} = \frac{-(-1) \pm \sqrt{(-1)^2 - 4 \cdot 3 (-1)}}{2 \cdot 3} = \frac{1 \pm \sqrt{1 + 12}}{6} = \frac{1 \pm \sqrt{13}}{6}$$

$$\boxed{x_3 = \frac{1 - \sqrt{13}}{6}, \quad x_4 = \frac{1 + \sqrt{13}}{6}}$$

③ (tr 3)

$$\sqrt{x+1}+\sqrt{x+2}=\sqrt{x+3}\ /^2$$

$$x+1\geq 0 \Longrightarrow \quad x\geq -1$$ da bismo dobili realna rješenja!

$$x+2\geq 0$$

$$x+3\geq 0$$

$$\left(\sqrt{x+1}+\sqrt{x+2}\right)^2=\left(\sqrt{x+3}\right)^2$$

$$x+1+2\sqrt{(x+1)(x+2)}+x+2=x+3$$

$$2x+\cancel{3}+2\sqrt{(x+1)(x+2)}=x+\cancel{3}$$

$$x+2\sqrt{(x+1)(x+2)}=0$$

$$2\sqrt{(x+1)(x+2)}=-x\ /^2$$

$$4(x+1)(x+2)=x^2$$

$$4(x^2+2x+x+2)=x^2$$

$$4x^2+8x+4x+8=x^2$$

$$3x^2+12x+8=0$$

$$x_{1,2}=\frac{-12\pm\sqrt{12^2-4\cdot3\cdot8}}{2\cdot3}=\frac{-12\pm\sqrt{144-96}}{6}$$

$$x_{1,2}=\frac{-12\pm\sqrt{48}}{6}=-2\pm\frac{\overset{2}{\cancel{4}}\sqrt{3}}{\underset{3}{\cancel{6}}}=-2\pm\frac{2}{3}\sqrt{3}$$

$$\boxed{\underbrace{\cancel{x_1=-2-\frac{2}{3}\sqrt{3}}}_{x\geq -1},\quad x_2=-2+\frac{2}{3}\sqrt{3}}$$

prihvaćamo!

④ (Pr 5)

$$x^2+mx+n=0$$

$$x_1+x_2=-2 \quad (1) \qquad m, n = ?$$

$$x_1-x_2=5 \quad (2)$$

$$x_{1,2}=\frac{-m\pm\sqrt{m^2-4n}}{2} \quad \nearrow x_1=-\frac{m-\sqrt{m^2-4n}}{2} \quad \searrow x_2=-\frac{m+\sqrt{m^2-4n}}{2}$$

(1), (2)

$$x_1+x_2=\frac{-m+\sqrt{m^2-4n}}{2}+\left(\frac{-m-\sqrt{m^2-4n}}{2}\right)=-2$$

$$=-\frac{m}{2}-\frac{m}{2}=-m \Rightarrow \boxed{m=2}$$

$$x_1-x_2=5$$

$$x_1-x_2=\frac{-m+\sqrt{m^2-4n}}{2}-\left(-\frac{m-\sqrt{m^2-4n}}{2}\right)=\sqrt{m^2-4n}/2$$

$$m^2-4n=5^2=25$$

$$m^2-4n-25=0$$

$$4-4n-25=0$$

$$4n=-21 \Rightarrow \boxed{n=-\frac{21}{4}}$$

⑤ (Pr 7)

$$x^2y+xy^2=20$$

$$\frac{1}{x}+\frac{1}{y}=\frac{5}{4}$$

$$xy(x+y)=20 \quad (3)$$

$$\frac{x+y}{xy}=\frac{5}{4}$$

$(\cdot)$ → „pomnožimo ih"

$$\Rightarrow \overset{1}{\cancel{xy}}(x+y)\cdot\frac{x+y}{\underset{1}{\cancel{xy}}}=\overset{5}{\cancel{20}}\cdot\frac{5}{\underset{1}{\cancel{4}}}=25$$

$$(x+y)^2 = 25 \quad \sqrt{\ }$$

s obzirom da je potencija

$$\Rightarrow \quad \text{I)}\ x+y=5 \xrightarrow{\text{u } (3)} xy(x+y)=20$$

$$\text{II)}\ x+y=-5$$

$$xy\cdot 5 = 20$$

$$xy = 4$$

$$\Downarrow$$

$$xy=-4$$

I) $x+y=5$

$xy=4 \Rightarrow y=\frac{4}{x}$

$$x+\frac{4}{x}=5$$

$$\frac{x^2+4}{x}=5$$

$$x^2-5x+4=0$$

$$x_{1,2}=\frac{5\pm\sqrt{25-4\cdot 4}}{2}=\frac{5\pm 3}{2}$$

$$\boxed{x_1=1\ ,\ x_2=4}$$

II) $x+y=-5$

$xy=-4 \Rightarrow y=-\frac{4}{x}$

$$x+\left(-\frac{4}{x}\right)=-5$$

$$\frac{x^2-4}{x}=-5$$

$$x^2+5x-4=0$$

$$x_{3,4}=\frac{-5\pm\sqrt{25-4\cdot(-4)}}{2}=\frac{-5\pm\sqrt{41}}{2}$$

$$\boxed{\begin{array}{l} x_3=\frac{-5-\sqrt{41}}{2} \\ x_4=\frac{-5+\sqrt{41}}{2} \end{array}}$$

⑥ (Pr 12) $f(x) = -x^2+2x+3$

1) ekstremna vrijednost funkcije:

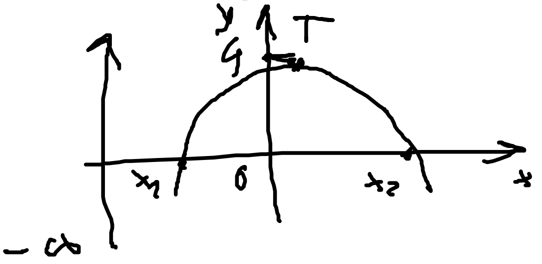
$a<0 \quad a=-1 \quad \cap$

$$\boxed{x_0 = -\frac{b}{2a} = -\frac{2}{2\cdot(-1)} = 1}$$

tjeme funkcije (parabola) $T(0,1)$

$$\boxed{f(x_0=1) = -1^2+2\cdot1+3 = -1+5 = 4}$$

2) $\boxed{y=f(x)\in(-\infty,4]}$



3) Nultočke funkcije f:

$-x^2+2x+3=0$

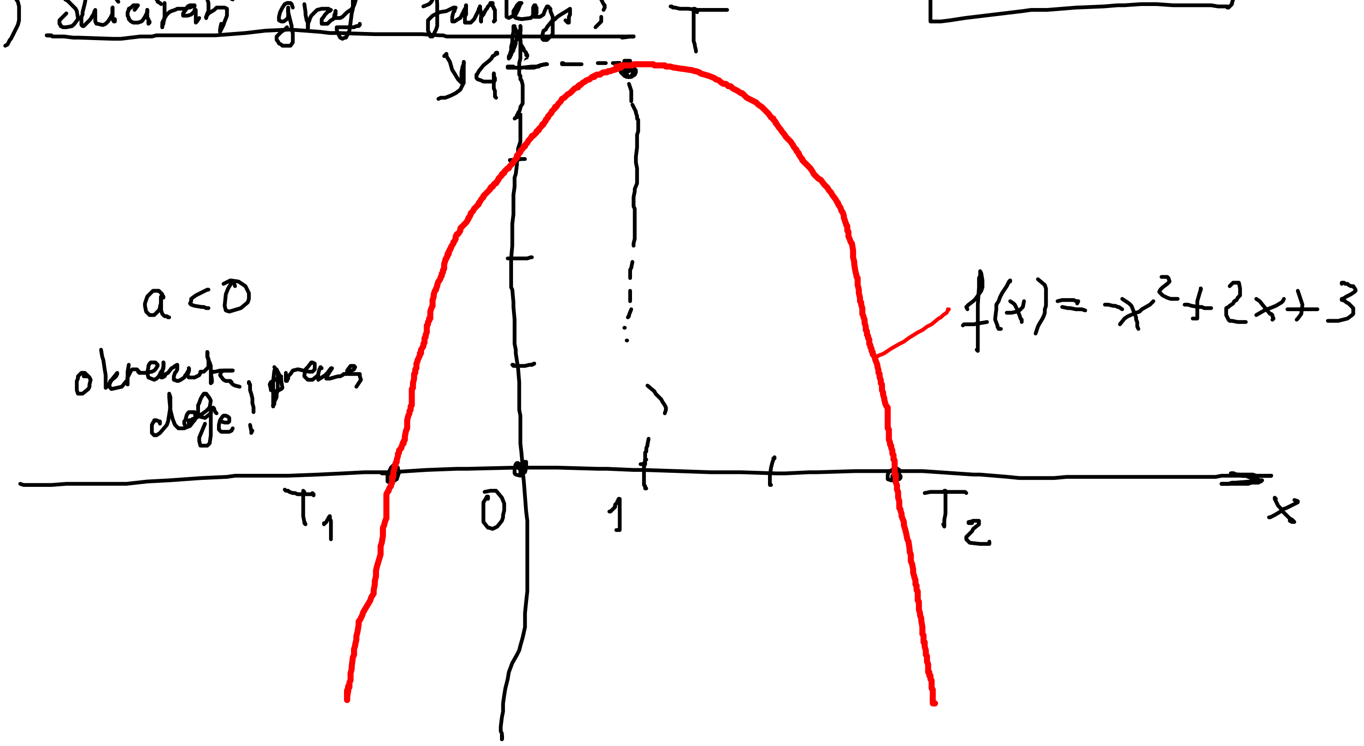
$$x_{1,2} = \frac{-2\pm\sqrt{2^2-4\cdot(-1)\cdot3}}{2\cdot(-1)} = \frac{-2\pm4}{-2} \begin{cases} \nearrow x_1=-1 \\ \searrow x_2=3 \end{cases}$$

$f(x_1) = -(-1)^2+2\cdot(-1)+3 = 0$

$f(x_2) = -3^2+2\cdot3+3 = 0$

$$\boxed{\begin{array}{c} T_1(-1,0) \\ T_2(3,0) \end{array}}$$

4) Skicirati graf funkcije:



⑦ (Pr 18)

Ⓟ ... $y = -x^2 + x + 1$

ⓟ ... $y = 3x + 11$ $\qquad t \parallel p \Rightarrow \underline{a_t = a_p = 3}$

$T(x,y) = ?$

└ diralište parabole i tangente

$t \; .... \; y_t = f(x) = ?$ – jednadžba tangente

$y_t = 3x + b_T$? → kada se sijeku parabola i tangenta – u jednoj točki – imamo diralište

$D = 0 = b^2 - 4ac$

$$\left.\begin{array}{l} y_p = -x^2 + x + 1 \\ y_t = 3x + b_T \end{array}\right\} \quad \begin{array}{l} y_p = y_t \\ -x^2 + x + 1 = 3x + b_T \\ -x^2 - 2x - b_T + 1 = 0 \quad (1) \end{array}$$

$$D = (-2)^2 - 4 \cdot (-1)(-b_T + 1) = 0$$

$$4 - 4b_T + 4 = 0$$

$$-4b_T = -8 \Rightarrow \boxed{b_T = 2}$$

$$\boxed{y_t = 3x + 2} \; ... \; ⓣ$$

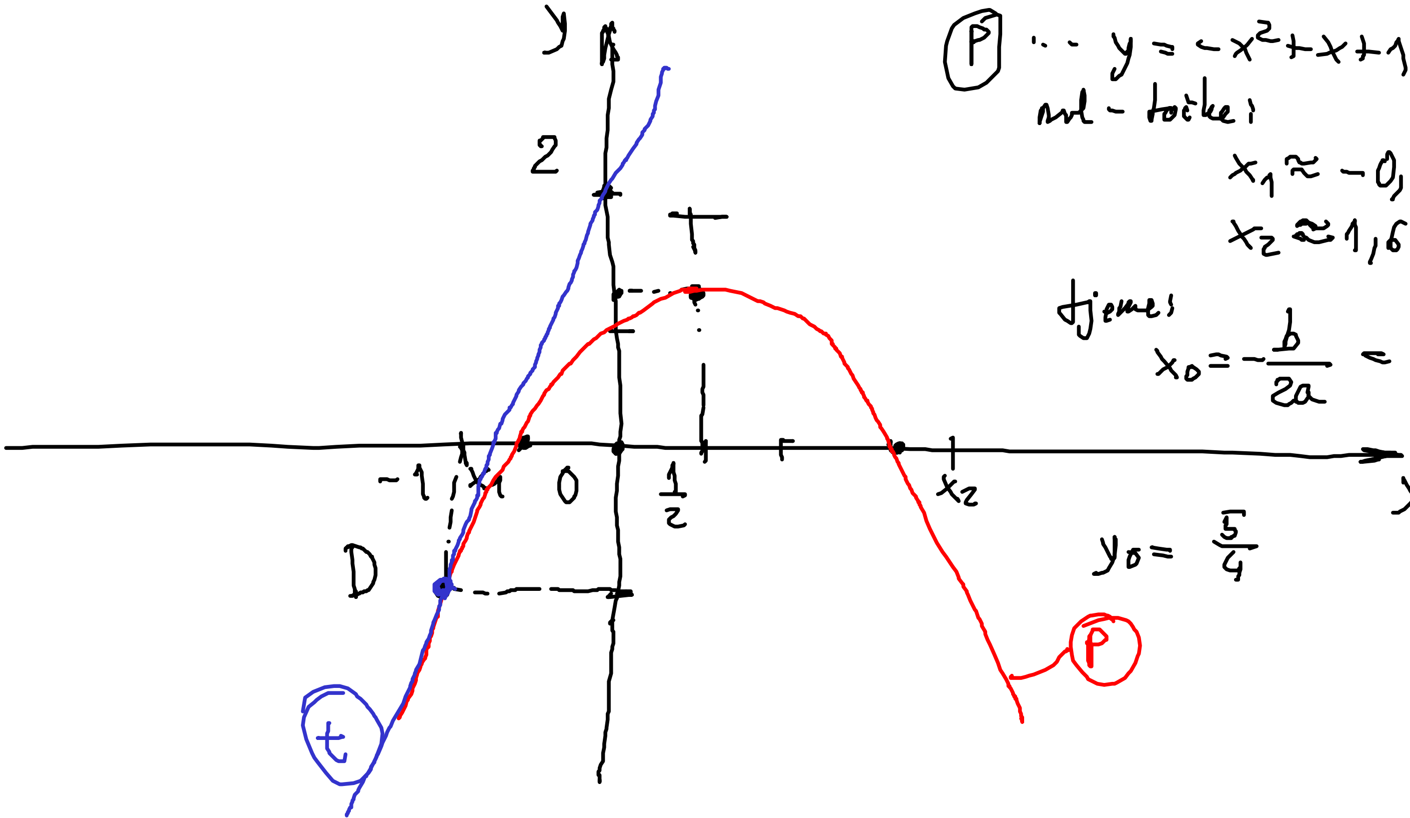
Koordinate dirališta i

$b_T = 2 \rightarrow (1)$ $\qquad x^2 + 2x + 1 = 0$

$$x_{1,2} = \frac{-2 \pm \sqrt{2^2 - 4 \cdot 1}}{2} = \frac{-2 \pm 0}{2} = -1$$

$$\boxed{x_1 = -1}$$

$$\left.\boxed{f(x_1 = -1) = y_t(x_1 = -1) = 3 \cdot (-1) + 2 = -1}\right\} \boxed{D(-1, -1)}$$



Ⓟ $\ldots\ y = -x^2+x+1$

nul - točke:

$x_1 \approx -0,62$

$x_2 \approx 1,62$

tjeme:

$x_0 = -\frac{b}{2a} = \frac{1}{2}$

$y_0 = \frac{5}{4}$

⑧ (zadaci - 26)

1) $$\frac{2x-1}{x^2-x+1} < 1$$

$$\frac{2x-1}{x^2-x+1} - 1 < 0$$

$$\frac{2x-1-(x^2-x+1)}{x^2-x+1} < 0$$

$$\frac{-x^2+3x-2}{x^2-x+1} < 0$$

y_1 y_2

$x_{1,2} = \frac{3}{2} \pm \frac{1}{2}$ → $x_1 = 1$, → $x_2 = 2$

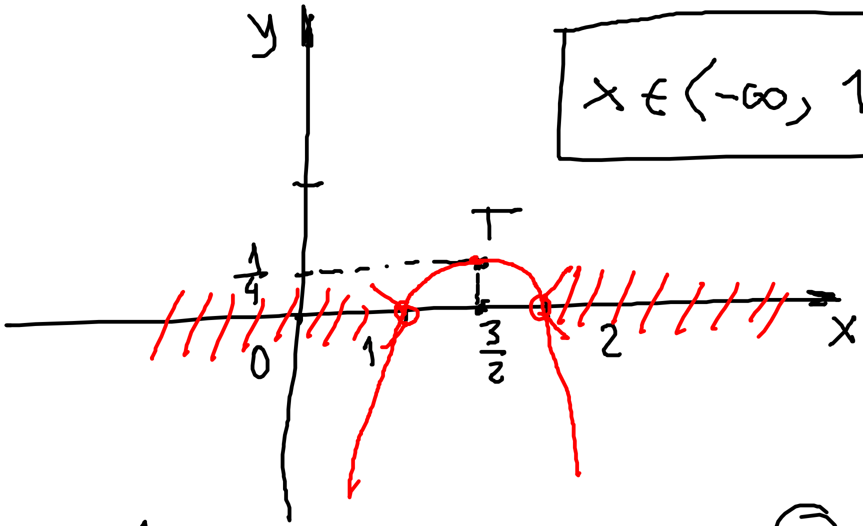
→ $D<0$ nema realnih rješenja

$$x_{1,2} = \frac{1 \pm \overbrace{\sqrt{1^2-4\cdot 1}}^{\sqrt{-3}}}{2}$$

tjeme:

$$x_0 = -\frac{b}{2a} = \frac{-3}{2\cdot(-1)} = \frac{3}{2}$$

$$y_0\left(x_0 = \frac{3}{2}\right) = \ldots = \frac{1}{4}$$



$$\boxed{x \in (-\infty, 1\rangle \cup \langle 2, +\infty\rangle}$$

2) $\frac{x-1}{x^2+3x-4} \geq 1$

$\frac{x-1}{x^2+3x-4} - 1 \geq 0$

$\frac{x-1-(x^2+3x-4)}{x^2+3x-4} \geq 0$

$\frac{-x^2-2x+3}{x^2+3x-4} \geq 0$ (P_1 – numerator, P_2 – denominator)

P_1

$x_{1,2}$ → $x_1 = -3$, → $x_2 = 1$

tjeme: $x_0 = -\frac{b}{2a} = -\frac{(-2)}{2\cdot(-1)}$

$x_0 = -1$

$-(-1)^2 - 2\cdot(-1) + 3 = -1 + 2 + 3 = 4$

$y_0 = 4$

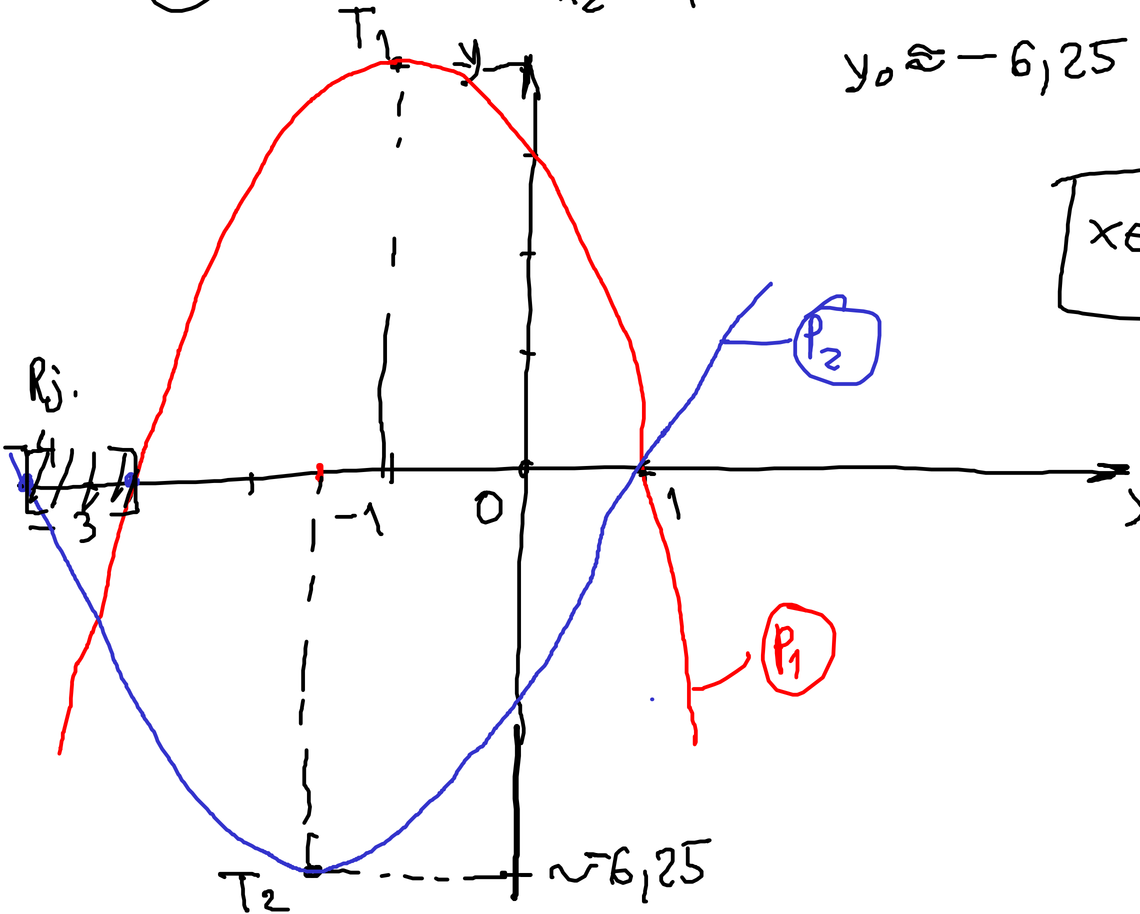
$\underline{T_1(-1, 4)}$

P_2

$x_{1,2}$ → $x_1 = -4$, → $x_2 = 1$

$x_0 = -\frac{b}{2a} = -\frac{3}{2}$

$y_0 \approx -6,25$ $\quad T_2\left(-\frac{3}{2}, -6.25\right)$



$$\boxed{x \in [-4, -3]}$$

⑨ (6 - ispit 1)

$$x+y=a$$

$$xy=4 \Rightarrow x=\frac{4}{y}$$

$a=?$ da sustav neme realnih rješenja

$$\frac{4}{y}+y=a \quad / \cdot y$$

$$4+y^2=ay$$

$$y^2-ay+4=0$$

$$D<0$$

$$D=b^2-4ac=(-a)^2-4\cdot 1\cdot 4=a^2-16<0$$

$$a^2<16$$

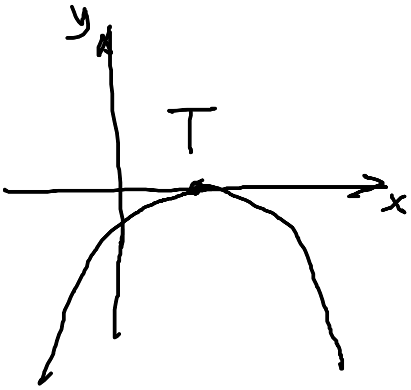
$$\boxed{\begin{matrix} a<4 \\ a>-4 \end{matrix}}$$

⑩

$$y=-2x^2+3x+c$$

$c=?$ - tjeme parabole na osi x

↓



$$D=0$$

$$D=b^2-4ac=0$$

$$3^2-4\cdot(-2)\cdot c=0$$

$$9+8c=0 \Rightarrow \boxed{c=-\frac{9}{8}}$$

8. EKSPONENCIJALNE I LOGARITAMSKE FUNKCIJE

① (Pr 4)

1) $\log_{\frac{1}{2}} x = -\frac{1}{2}$

$\log_a b = c \iff a^c = b$

$\Rightarrow \boxed{x = \left(\frac{1}{2}\right)^{-\frac{1}{2}} = \frac{1}{2^{-\frac{1}{2}}} = \frac{1}{\sqrt{2}} = \sqrt{2}}$

2) $\log_x 16 = \frac{4}{3} \Rightarrow x^{\frac{4}{3}} = 16 \quad /^{\frac{3}{4}}$

$\boxed{x = 16^{\frac{3}{4}} = \left(2^{\cancel{4}^{1}}\right)^{\frac{3}{\cancel{4}_{1}}} = 2^3 = 8}$

3) $\log_8 \sqrt[3]{16} = x$

$8^x = \sqrt[3]{16}$

$2^{3x} = \left(2^4\right)^{\frac{1}{3}} \Rightarrow 3x = \frac{4}{3} \quad /:3$

$\boxed{x = \frac{4}{9}}$

② (Pr 5)

1) $4^{-\log_2 7} = \left(2^2\right)^{-\log_2 7} = \left(\underbrace{2^{\log_2 7}}_{=7}\right)^{-2} = 7^{-2} = \frac{1}{7^2} = \boxed{\frac{1}{49}}$

2) $5^{2+\log_{25} 9} = 5^2 \cdot 5^{\log_{25} 9}$

$5 = \sqrt{25} = 25^{\frac{1}{2}}$

$= 25 \cdot 25^{\frac{1}{2} \cdot \log_{25} 9} = 25 \cdot \left(\underbrace{25^{\log_{25} 9}}_{=9}\right)^{\frac{1}{2}} = 25 \cdot 9^{\frac{1}{2}} = 25 \cdot \sqrt{9}$

$= 25 \cdot 3 = \boxed{75}$

3) $\left(\frac{1}{3}\right)^{-\log_{\sqrt{3}}12+3\log_9 4}$

$$(3^{-1})=\left(\left(\sqrt{3}\right)^{-2}\right)^{-1}=\left(\sqrt{3}\right)^{-(-2)}$$

$$=\left(\frac{1}{3}\right)^{-\log_{\sqrt{3}}12}\cdot\left(\frac{1}{3}\right)^{3\log_9 4}=\left(\sqrt{3}\right)^{-(-2)\sqrt{3}\,12}\cdot 3^{(-1)3\log_9 4}$$

$$=\left(\sqrt{3}\right)^{2\log_{\sqrt{3}}12}\cdot \underbrace{3}_{9^{\frac{1}{2}}}{}^{-3\log_9 4}=\underbrace{\left(\left(\sqrt{3}\right)^{\log_{\sqrt{3}}12}\right)^2}_{12^2}\cdot\underbrace{9^{-\frac{3}{2}\log_9 4}}_{4^{-\frac{3}{2}}}$$

$$\underbrace{4}_{}{}^{-\frac{3}{2}}=\left(2^2\right)^{-\frac{3}{2}}=\frac{1}{8}$$

$$=\frac{144}{8}=\boxed{18}$$

③ (Pr 8)

$$\frac{\log_3 2=m}{\log_{18}36=?}$$

$$\log_a c=\frac{\log_b c}{\log_b a}$$

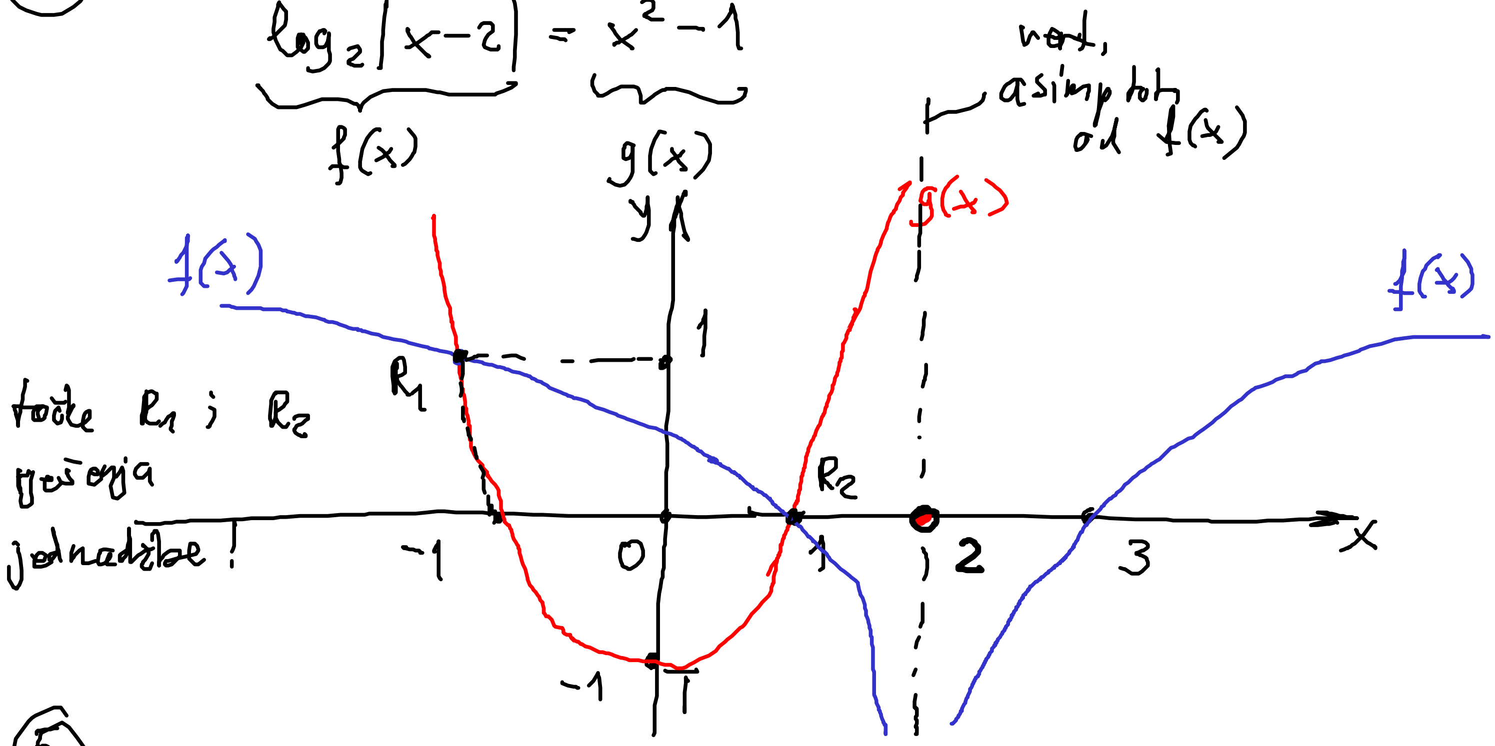
$$\boxed{\log_{18}36=}\ \frac{\log_3 36}{\log_3 18}=\frac{\log_3(9\cdot4)}{\log_3(9\cdot2)}=\frac{\overbrace{\log_3 9}^{=2}+\log_3 \overset{2^2}{4}}{\underbrace{\log_3 9}_{=2}+\log_3 2}$$

$$=\frac{2+\log_3 2^2}{2+\log_3 2}=\frac{2+2\underbrace{\log_3 2}_{=m}}{2+\underbrace{\log_3 2}_{=m}}$$

$$=\boxed{\frac{2(1+m)}{2+m}}$$

④ (Pr 11)

$$\underbrace{\log_2|x-2|}_{f(x)}=\underbrace{x^2-1}_{g(x)}$$



⑤

$$27^{2-x}\cdot\sqrt[3]{9^{2x+1}}=\left(\frac{1}{3}\right)^{x-4}$$

$$\left(3^3\right)^{2-x}\cdot\left(\left(3^2\right)^{2x+1}\right)^{\frac{1}{3}}=\left(\left(3\right)^{-1}\right)^{x-4}$$

$$3^{6-3x}\cdot\left(3^{4x+2}\right)^{\frac{1}{3}}=3^{-x+4}$$

$$3^{6-3x+\frac{4x+2}{3}}=3^{-x+4}$$

$$\Rightarrow 6-3x+\frac{4x+2}{3}=-x+4$$

$$\frac{18-9x+4x+2}{3}=-x+4$$

$$\frac{-5x+20}{3}=-x+4 \quad /\cdot 3$$

$$-5x+20=-3x+12$$

$$-2x=-8 \Rightarrow \boxed{x=4}$$

⑥ (Pr 14)

$$\underbrace{5^x}_{\hookrightarrow (\sqrt{5})^{2x}} + 5 = 6\cdot(\sqrt{5})^x$$

$$(\sqrt{5})^{2x} - 6\cdot(\sqrt{5})^x + 5 = 0$$

$$\underline{t = (\sqrt{5})^x \text{ – supstitucija}}$$

$$t^2 - 6\cdot t + 5 = 0$$

$$t_{1,2} \begin{array}{l} \nearrow t_1 = 1 \\ \searrow t_2 = 5 \end{array}$$

I) za $t_1 = 1$

$$(\sqrt{5})^x = 1 \Rightarrow \boxed{x_1 = 0}$$

II) za $t_2 = 5$

$$(\sqrt{5})^x = 5 \Rightarrow \boxed{x_2 = 2}$$

⑦ (Pr 15)

$$\log(0.1x^2)\cdot\log\left(\frac{x}{10}\right) = 3$$

$$(\underbrace{\log 0.1}_{=-1} + \underbrace{\log x^2}_{2\log x})\cdot(\log x - \underbrace{\log 10}_{=1}) = 3$$

$$(-1 + 2\log x)\cdot(\log x - 1) = 3$$

$$-\log x + 2\log^2 x + 1 - 2\log x = 3$$

$$2\log^2 x - 3\log x - 2 = 0$$

$$\underline{t = \log x \text{ – supstitucija}}$$

$$2t^2-3t-2=0$$

$$\begin{tikzcd} & t_1=-\frac{1}{2} \\ t_{1,2} \arrow[ur] \arrow[dr] & \\ & t_2=2 \end{tikzcd}$$

I) za $t_1=-\frac{1}{2}$ $\quad \log x=-\frac{1}{2}$ $\quad \boxed{x_1=\frac{\sqrt{10}}{10}}$

II) za $t_2=2$ $\quad \log x=2$ $\quad \boxed{x_2=100}$

⑧ (Pr 1M)

$$3^{x-2}\cdot 2^{y-4}=144 \qquad (1)$$

$$1+\log_2 x=\log_2 y \qquad (2)$$

iz (2):

$$\log_2 x=\log_2 y-1$$

$$\log_2 x-\log_2 y=-1 \quad /\cdot(-1)$$

$$\log_2 y-\log_2 x=1$$

$$\log_2\left(\frac{y}{x}\right)=1 \Rightarrow \underline{y=2x} \rightarrow (1)$$

$$3^{x-2}\cdot 2^{2x-4}=144$$

$$3^{x-2}\cdot 2^{2(x-2)}=3^{x-2}\cdot 4^{x-2}=12^{x-2}$$

$$12^{x-2}=144=12^2 \Rightarrow x-2=2$$

$$\boxed{x=4}$$

$$\boxed{y=8}$$

⑨ (Pr 18) $\overbrace{0.8^{2x-3}}^{f(x)} \le \left(\frac{\sqrt{5}}{2}\right)^{x+1}$ — g(x)

$$\left(\frac{\cancel{8}^{\,4}}{\cancel{10}_{\,5}}\right)^{2x-3} \le \left(\frac{5^{\frac{1}{2}}}{4^{\frac{1}{2}}}\right)^{x+1} = \left(\frac{4}{5}\right)^{-\frac{1}{2}(x+1)}$$

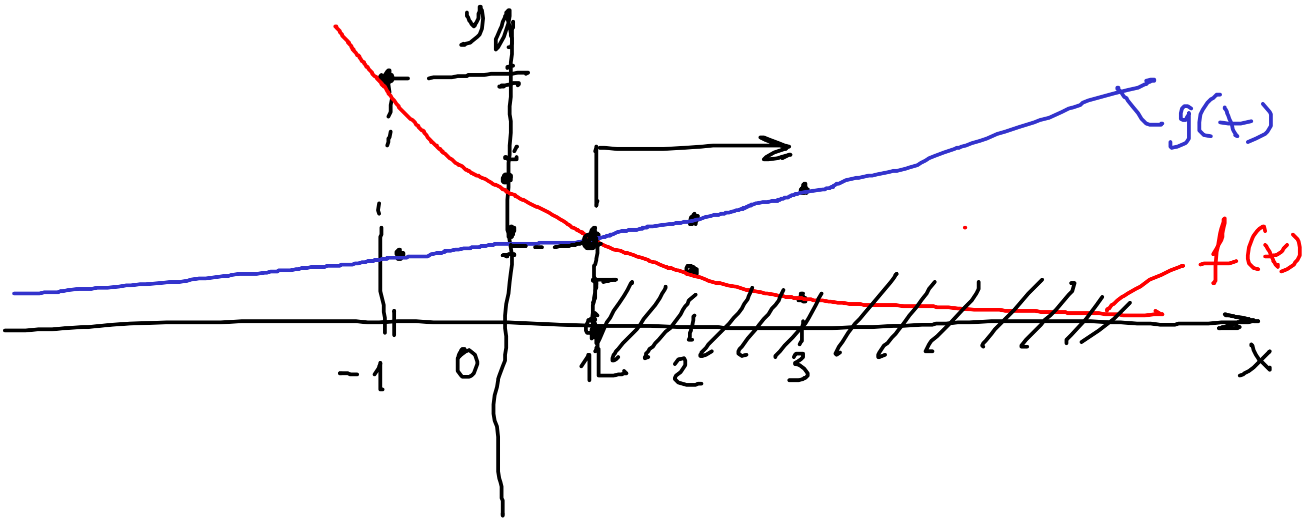
$$\Rightarrow 2x-3 \ge -\frac{1}{2}(x+1)$$

$a = \frac{4}{5}$ – baza < 1 $\quad \le \rightarrow \ge$

$$\frac{5}{2}x \ge \frac{5}{2}$$

$$x \ge 1$$

$$x \in [-1, +\infty\rangle$$



$\boxed{f(x)}$

x	-1	0	1	2	3
f(x)	3,05	1,95	1,25	0,8	0,512

g(x)

x	-1	0	1	2	3
g(x)	1	1,12	1,25	1,4	1,56

⑩ (Pr 19)

$$0.4^{x} - 2.5^{x+1} > 1.5$$

$$\left(\frac{4}{10}\right)^{x} - \left(\frac{25}{10}\right)^{x+1} > \frac{15}{10}$$

$$\left(\frac{2}{5}\right)^{x} - \left(\frac{5}{2}\right)^{x+1} > \frac{3}{2}$$

$t=\left(\frac{2}{5}\right)^{x}$ - supstitucija

$$t - \frac{5}{2t} > \frac{3}{2} \quad /\cdot 2t$$

$$2t^{2} - 5 > 3t$$

$$2t^{2} - 3t - 5 > 0$$

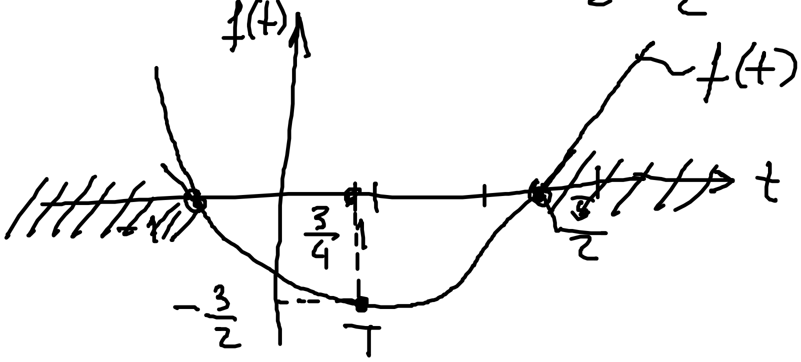
$t_1 = -1$

$t_2 = \frac{5}{2}$

$t_0 = -\frac{b}{2a} = \frac{3}{4}$

$f(t_0) = -\frac{3}{2}$

$a > 0 \ \cup$



1) $t_1 = -1$

$\left(\frac{2}{5}\right)^{x} = t$

$\left(\frac{2}{5}\right)^{x} < -1$ ne može biti!

II) $\left(\frac{2}{5}x\right) = t = \frac{5}{2}$

$\left(\frac{2}{5}\right)^{x} > \left(\frac{2}{5}\right)^{-1}$

$a < 0$

$\boxed{x < --1}$

$\boxed{x \in \langle -\infty, -1 \rangle}$ ✓

9. FUNKCIJE

① (Pr 2)

$$\frac{f(1-x)=1-x^2 \quad , \quad x\in R}{f(1+x)=?}$$

$t=1-x$ – supstitucija

$\Rightarrow x=1-t \,/^2$

$x^2=1-2t+t^2$

$$f(t)=1-(1-2t+t^2)=\cancel{1}-\cancel{1}+2t-t^2=-t^2+2t$$

$$f(1+x)=-(1+x)^2+2(1+x)=-(1+2x+x^2)+2+2x$$

$$=-1-\cancel{2x}-x^2+2+\cancel{2x}=\boxed{-x^2+1}$$

② (Pr 3)

$$\frac{\frac{1}{2}f(x)+2f\left(\frac{1}{x}\right)=x \quad , \quad x\in D_f}{f(2)=?}$$

za $x=2$ $\quad \frac{1}{2}f(2)+2f\left(\frac{1}{2}\right)=2 \quad (1)$

za $x=\frac{1}{2}$ $\quad \frac{1}{2}f\left(\frac{1}{2}\right)+2f(2)=\frac{1}{2} \quad (2) \;/\cdot(-4)$

$$\left.\begin{aligned}\frac{1}{2}f(2)+\cancel{2f\left(\tfrac{1}{2}\right)}&=\cancel{2}\\ -\cancel{2f\left(\tfrac{1}{2}\right)}-8f(2)&=-\cancel{2}\end{aligned}\right\}(+)$$

$$\left(-8+\frac{1}{2}\right)f(2)=0 \Rightarrow \boxed{f(2)=0}$$

③ $f(x) = \frac{2^x - 3^x}{2^x + 3^x}$ parna ili neparna?

$f(-x) = -f(x)$ neparna

$f(-x) = f(x)$ parna

$$f(-x) = \frac{2^{-x} - 3^{-x}}{2^{-x} + 3^{-x}} \Big/ \cdot \frac{2^x \cdot 3^x}{2^x \cdot 3^x}$$

$$= \frac{\overbrace{2^{-x+x}}^{=1} \cdot 3^x - 2^x \cdot \overbrace{3^{x-x}}^{=1}}{\underbrace{2^{-x+x}}_{=1} \cdot 3^x + 2^x \cdot \underbrace{3^{x-x}}_{=1}} = \frac{3^x - 2^x}{3^x + 2^x}$$

$$= -\frac{2^x - 3^x}{2^x + 3^x} = -\left(\frac{2^x - 3^x}{2^x + 3^x}\right) = \boxed{-f(x)}$$ funkcija je neparna!

④ (Pr 8)

$$f(x) = (\sin 2x + \cos 2x)^2$$

1) Parnost / neparnost od f(x):

$f(-x) = (\cdot)$

$$f(x) = \underbrace{\sin^2 2x}_{} + \overbrace{2 \sin 2x \cos 2x}^{\sin 4x} + \underbrace{\cos^2 2x}_{} = 1 + \sin 4x$$

$= 1$

$f(-x) = 1 + \sin 4(-x) = 1 - \sin 4x$ funkcija nije ni parna ni neparna!

2) Periodičnost: periodična je!

$$f(x) = a \sin(bx + c)$$

$$\boxed{P = \frac{2\pi}{b} = \frac{2\pi}{4} = \frac{\pi}{2}}$$ Temeljni period funkcije!

3) Nul točke funkcije:

$$f(x)=0$$

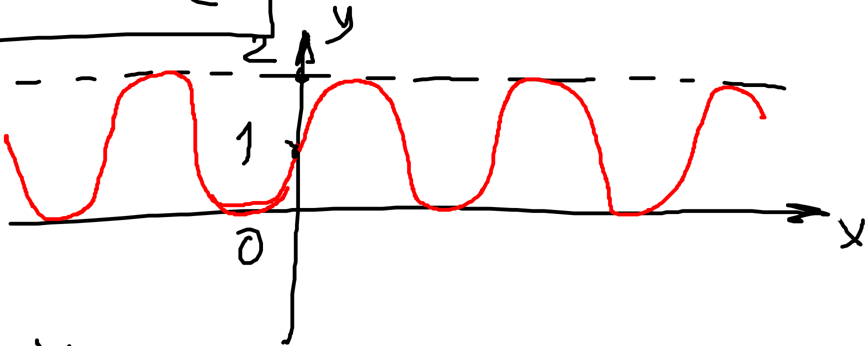
$$1+\sin 4x=0$$

$$\sin 4x=-1$$

$$4x=-\frac{\pi}{2}+k\cdot 2\pi \quad /:4$$

$$\boxed{x=-\frac{\pi}{8}+k\cdot\frac{\pi}{2}}$$

4) $f(x)\in[0,2]$



⑤ (Pr 11)

$$f(x)=\frac{x}{x-2}, \quad x\neq 2$$

$$y=\frac{x}{x-2} \Rightarrow x=y(x-2)$$

$$x=xy-2y$$

$$x-xy=-2y$$

$$x(1-y)=-2y$$

$$x=\frac{-2y}{1-y}=\frac{2y}{y-1}$$

- za $f(x)$ mora postojati jedinstveno rješenje, tada f ima svoj inverz $x=f^{-1}(y)$

$$f^{-1}(y)=\frac{2y}{y-1}$$

- zamjena imena nepoznanica x i y

$$y=f^{-1}(x)$$

$$\boxed{f^{-1}(x)=\frac{2x}{x-1}}$$

⑥ (Prg)

$$\frac{\begin{array}{l} f(x) = x^2 - x - 1 \\ g(x) = 2x + 1 \end{array}}{f \circ g(x) \leq g \circ f(x)}$$

$$f \circ g(x) = (2x+1)^2 - (2x+1) - 1 = 4x^2 + 4x + 1 - 2x - 1 - 1$$

$$= 4x^2 + 2x - 1$$

$$g \circ f(x) = 2(x^2 - x - 1) + 1 = 2x^2 - 2x - 2 + 1 = 2x^2 - 2x - 1$$

$$4x^2 + 2x - 1 \leq 2x^2 - 2x - 1$$

$$2x^2 + 4x \leq 0 \;/ : 2$$

$$x^2 + 2x \leq 0$$

$$x(x+2) \leq 0$$

$$\left.\begin{array}{l} x_1 \leq 0 \\ x_2 + 2 \leq 0 \\ x_2 \geq -2 \end{array}\right\} \boxed{x \in [-2, 0]}$$

⑦

$$\frac{f(8^{x-1}) = 4^{x+1}}{f^{-1}(x) = ?}$$

– naći inverz funkcije!

$$4^{x+1} = 4^{x+1-1+1} = 4^{x-1} \cdot 4^2 = 16 \cdot (2^2)^{x-1}$$

$$= 16 \cdot 2^{2(x-1)\frac{3}{3}} = 16 \cdot 2^{3(x-1)\frac{2}{3}} = 16 \cdot (8^{x-1})^{\frac{2}{3}} = f(8^{x-1})$$

$$f(x) = 16\, x^{\frac{2}{3}} = y$$

$$x = 16\, y^{\frac{2}{3}} = 16 \cdot \sqrt[3]{y^2} \;/^3$$

$$x^3 = 16^3 \cdot y^2 \Rightarrow y^2 = \frac{x^3}{16^3} \;/^{\frac{1}{2}}$$

$$y = \frac{x^{\frac{3}{2}}}{16^{\frac{3}{2}}} = \frac{x\sqrt{x}}{\left(2^{\cancel{4}_{2}}\right)^{\frac{3}{\cancel{2}_{1}}}} = \frac{1}{64}x\sqrt{x}$$

f definirana za sve realne brojeve, $\forall x \in R$

$$\boxed{f^{-1}(x) = \frac{1}{64}x\sqrt{x}}$$

⑧ (Pr 12) $f(x) = a \cdot \log_2 x + b$

$f(2) = 0$

$f(1) = 1$

$f^{-1}(x) = ?$

$$f(2) = 0 = a \cdot \overbrace{\log_2 2}^{=1} + b = a + b$$

$$f(1) = 1 = a \cdot \underbrace{\log_2 1}_{=0} + b = b \Rightarrow b = 1$$

$$a = -1$$

$$f(x) = -\log_2 x + 1$$

$$y = -\log_2 x + 1$$

$$\log_2 x = -y + 1$$

$$x = 2^{1-y}$$

$$\boxed{f^{-1}(x) = 2^{1-x}}$$