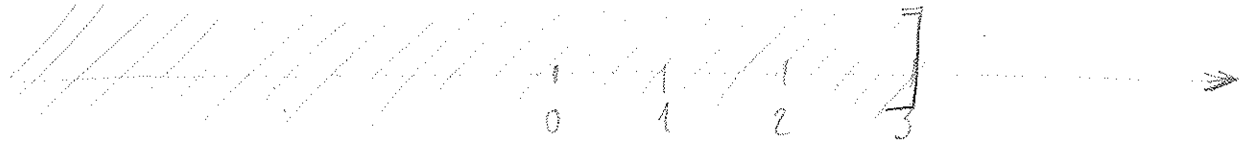
5. UREĐAJ NA SKUPU REALNIH BROJEVA

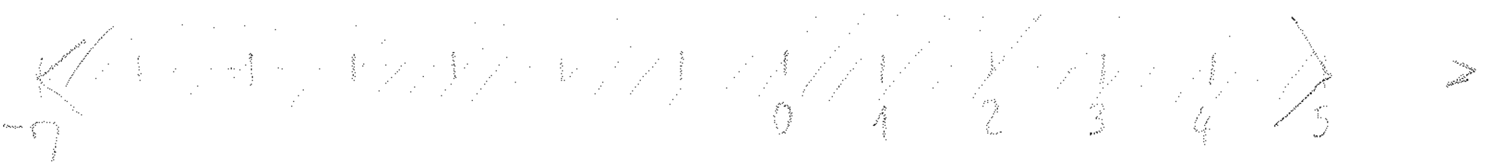
① (Primjer 3)
Dani su intervali $A = \langle -\infty, 3]$, $B = \langle -7, 5\rangle$, $C = [-1, +\infty\rangle$. Odredimo $A \cup B$, $B \cap C$ i $A \cap (B \cup C)$.

Skicirajmo intervale:

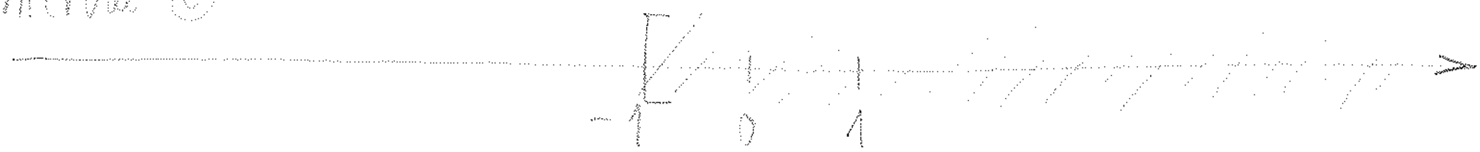
interval Ⓐ



interval Ⓑ



interval Ⓒ



$A \cup B = \langle -\infty, 5\rangle$

$B \cap C = [-1, 5\rangle$

$A \cap (B \cup C) = ?$

$B \cup C = \langle -7, +\infty\rangle$

$\langle -\infty, 3] \cap \langle -7, +\infty\rangle = \langle -7, 3]$

(2) (Primjer 4)

Riješimo sustav nejednadžbi:

$$\begin{cases} 1 - \dfrac{x-0.5}{5} < \dfrac{3x}{2} - 0,1 & (1) \\[2ex] \dfrac{4x-1}{3} - 0.25 > \dfrac{x}{4} & (2) \end{cases}$$

(1) $/ \cdot 10$

$$10 - 2(x-0.5) < 15x - 1$$

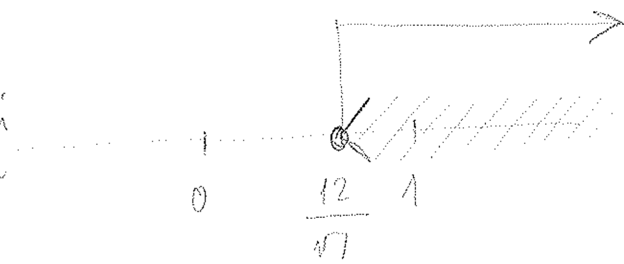
$$-2x - 15x < -1 - 10 - 1$$

$$-17x < -12 \quad / : (-17)$$

mijenja se predznak!

$$x > \frac{12}{17}$$

otvoreni interval



$$\left\langle \frac{12}{17}, +\infty \right\rangle$$

(2) $/ \cdot 12$

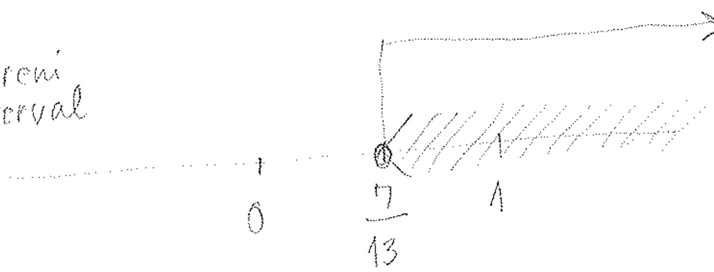
$$4(4x-1) - 0,25 \cdot 12 > 3x$$

$$16x - 4 - 3 > 3x$$

$$13x > 7$$

otvoreni interval

$$x > \frac{7}{13}$$



$$\left\langle \frac{7}{13}, +\infty \right\rangle$$

Rješenje ovih dviju nejednadžbi je presjek ovih dvaju dobivenih intervala

$$\boxed{\left\langle \frac{7}{13}, +\infty \right\rangle \cap \left\langle \frac{12}{17}, +\infty \right\rangle = \left\langle \frac{12}{17}, +\infty \right\rangle}$$

③ (Primjer 5)

Riješimo nejednadžbu:

$$\frac{x+1}{2x-5} \geq 1$$

$$\frac{x+1}{2x-5} - 1 \geq 0$$

$$\frac{x+1-(2x-5)}{2x-5} \geq 0$$

$$\frac{-x+6}{2x-5} \geq 0$$

→ to je ispunjeno u $\underline{\text{II}}$ slučaja:

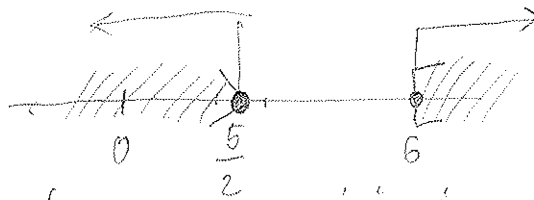
I) $-x+6 \geq 0 \Rightarrow x \leq 6$

$\quad 2x-5 > 0 \Rightarrow x > \frac{5}{2}$

$\left\langle \frac{5}{2}, 6 \right]$

II) $-x+6 \leq 0 \Rightarrow x \geq 6$

$\quad 2x-5 < 0 \Rightarrow x < \frac{5}{2}$



sustav nema rješenja

samo manje ili veće, u slučaju = neodređeni oblik $\frac{0}{0}$

$$\boxed{\left\langle \frac{5}{2}, 6 \right]}$$

④ Riješimo nejednadžbu (Primjer 6)

$$1 - \frac{2}{x-1} < \frac{2x}{x+1}$$

$$1 - \frac{2}{x-1} - \frac{2x}{x+1} < 0$$

$$\frac{x^2-1-2(x+1)-2x(x-1)}{(x-1)(x+1)} < 0$$

$$\frac{x^2 - 1 - \cancel{2x} - 2 - 2x^2 + \cancel{2x}}{(x-1)(x+1)} < 0$$

$$\frac{-x^2 - 3}{(x-1)(x+1)} < 0$$

$-x^2 - 3 < 0$ uvijek je negativan

mora biti $\rightarrow x^2 - 1 > 0$

$x^2 > 1$

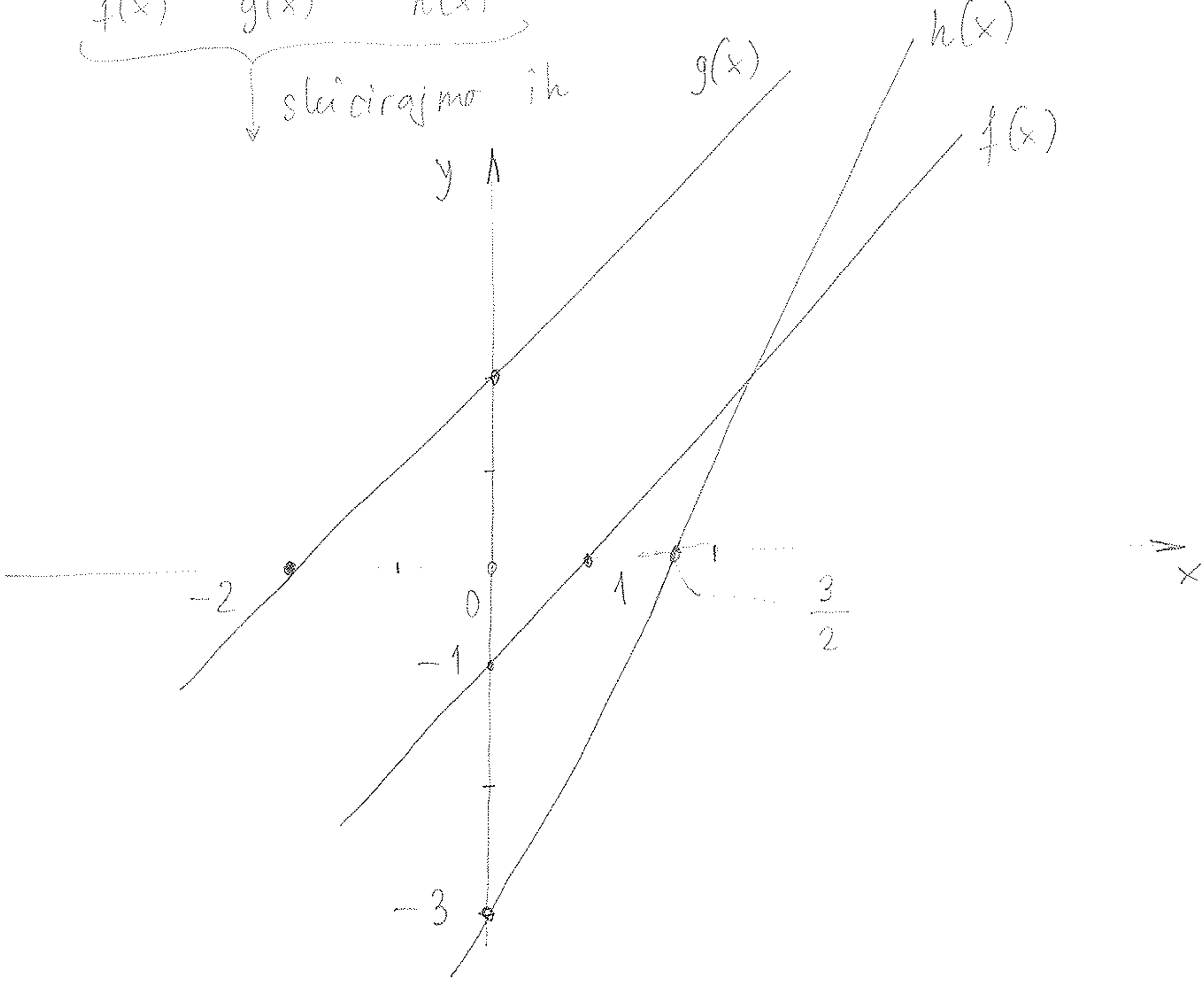
$|x| > 1$

$$\boxed{\langle -\infty, -1\rangle \cup \langle 1, +\infty\rangle}$$

5 (Primjer 7 - 1)

Riješimo nejednadžbu:

$$\underbrace{\underbrace{(x-1)}_{f(x)}\underbrace{(x+2)}_{g(x)}\underbrace{(2x-3)}_{h(x)}}_{\text{skicirajmo ih}} < 0$$



Promatramo intervale:

za $x < -2 \Rightarrow f(x), g(x), h(x) < 0$ ✓

za $-2 < x < 1 \Rightarrow \left.\begin{array}{l} g(x) > 0 \\ h(x), f(x) < 0 \end{array}\right\} f(x) \cdot g(x) \cdot h(x) >$

—

za $1 < x < \frac{3}{2} \Rightarrow \left.\begin{array}{l} g(x), f(x) > 0 \\ h(x) < 0 \end{array}\right\} f(x) \cdot g(x) \cdot h(x) <$

✓

za $x > \frac{3}{2} \Rightarrow f(x), g(x), h(x) > 0$ —

Rješenje je unija intervala (skup):

$$\boxed{\langle -\infty, -2\rangle \cup \langle 1, \tfrac{3}{2}\rangle}$$

⑥ (Primjer 10)

Riješimo nejednadžbu $|3x-1| \geq \frac{1}{2}$

I) $|3x-1| < 0 \Rightarrow -(3x-1) \geq \frac{1}{2} \quad / \cdot (-$

$3x - 1 \leq -\frac{1}{2}$

$3x \leq -\frac{1}{2} + 1 = \frac{1}{2}$

$x \leq \frac{1}{6}$

$\langle -\infty, \frac{1}{6}]$

II) $|3x-1| > 0$

$\Rightarrow (3x-1) \geq \frac{1}{2}$

$3x \geq \frac{1}{2} + 1 = \frac{3}{2}$

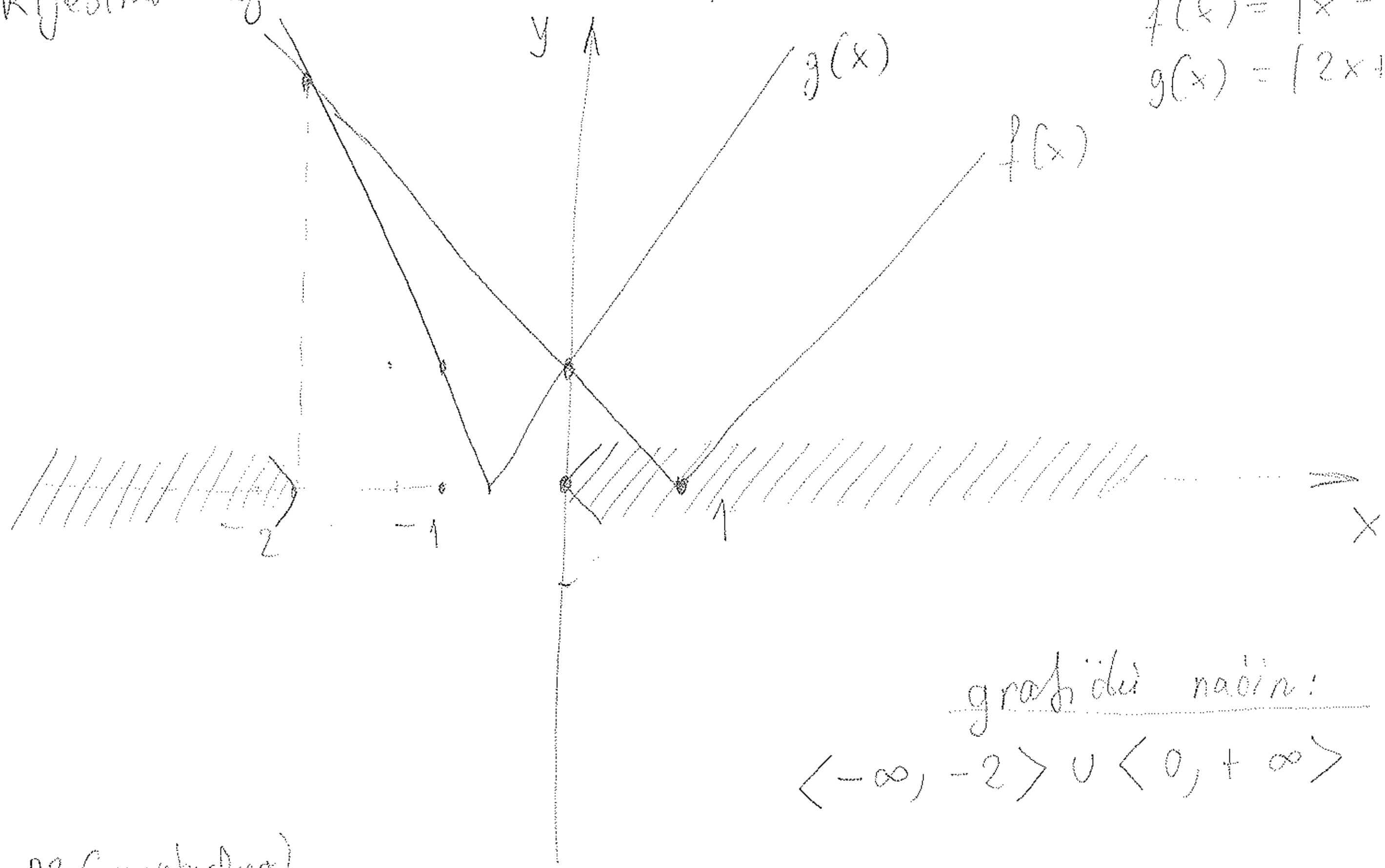
$\boxed{x \geq \frac{3}{6} = \frac{1}{2}}$ $\quad [\frac{1}{2}, +\infty\rangle$

Rješenje je unija intervala (skup):

$$\boxed{\langle -\infty, \tfrac{1}{6}] \cup [\tfrac{1}{2}, +\infty\rangle}$$

(7) (Primjer 11)
Riješimo nejednadžbu $|x-1| < |2x+1|$

$f(x) = |x-1|$
$g(x) = |2x+1|$



grafički način:
$\langle -\infty, -2 \rangle \cup \langle 0, +\infty \rangle$

DZ (eventualno)

~~(8)~~ (Zadaci - 8)
Odredite sve vrijednosti realnog parametra a za koje je rješenje jednadžbe

$$1 - \frac{a}{2} = \frac{1-ax}{x+2}$$ manje od 1

(8) (Zadaci - 21)
Razlomak $\frac{x|x-4|-4}{1-|1-x|}$ zapiši bez oznaka apsolutne vrijednosti uz uvjet $1 < x < 4$.

$1 < x < 4 \Rightarrow x > 0$ pozitivan $\Rightarrow$ razmak > 0

I) $\frac{-x(x-4)-4}{1-(-(1-x))} = \frac{-x^2+4x-4}{1+1-x} = -\frac{(x-2)(x-2)}{-x+2}$

$= -\frac{\cancel{(x-2)}(x-2)}{-\cancel{(x-2)}} = \boxed{x-2}$

Zadaci - 5

12

$$||x-1|-1|$$

$$x=\sqrt{2}-1$$

$$||\sqrt{2}-1-1|-1|$$

$$||\sqrt{2}-2|-1| = |-\sqrt{2}+2-1|$$

$$= |-\sqrt{2}+1| = \sqrt{2}-1$$

13

$$||x-2|-2|$$

$$x=1-\sqrt{3}$$

$$||1-\sqrt{3}-2|-2|$$

$$||-1-\sqrt{3}|-2|$$

$$|1+\sqrt{3}-2| \qquad |\sqrt{3}-1|$$

$$-\sqrt{3}+1$$

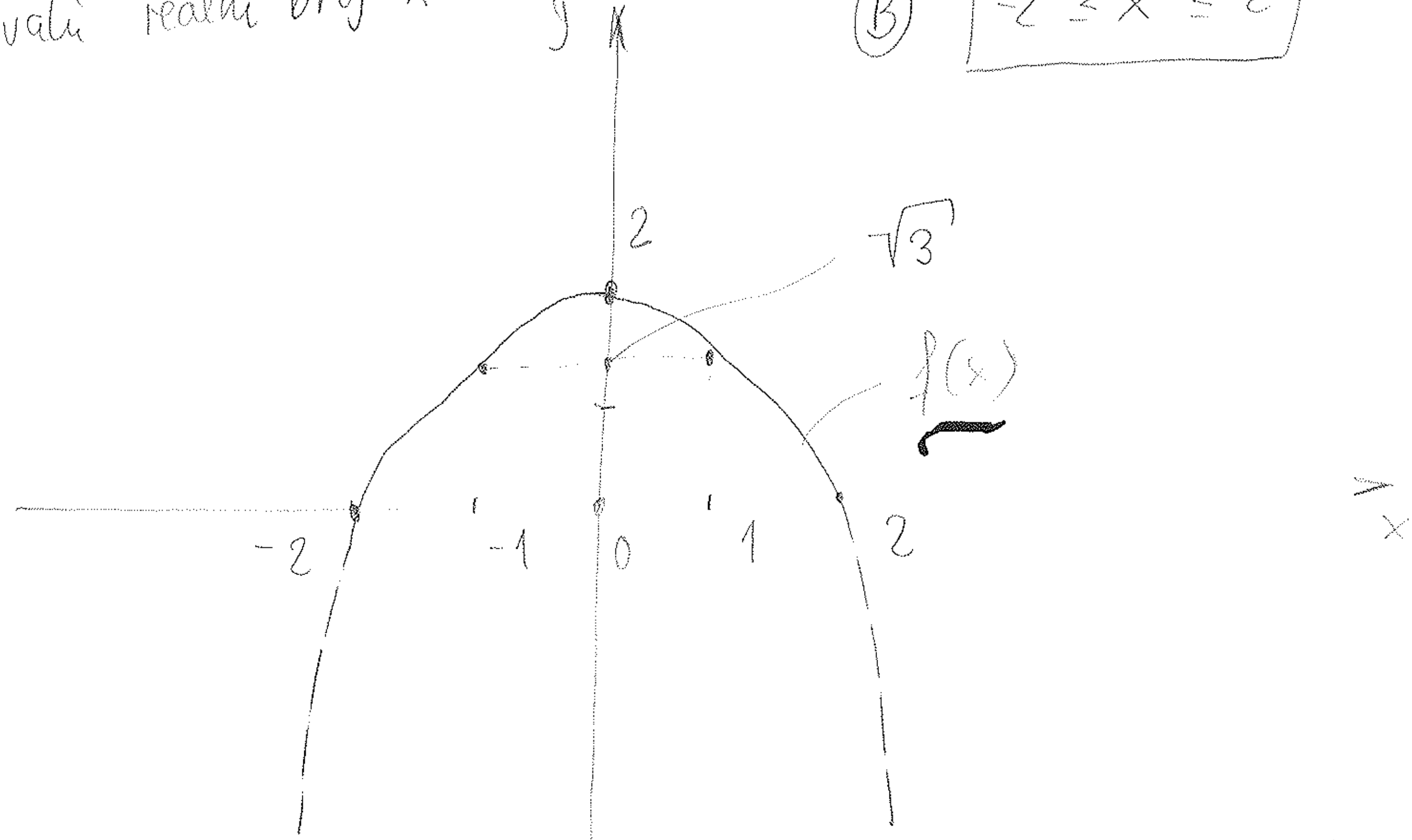
II). $\dfrac{x(x-4)-4}{1-1+x} = \dfrac{x^2-4x-4}{x}$

za $x=3$ $\dfrac{9-12-4}{3} < 0$

$-$

9 (ispit 1 - 4)
Realna funkcija $f(x)=\sqrt{4-x^2}$ definirana je za svaki realni broj x:

B $\boxed{-2 \le x \le 2}$



10 Nejednadžba $2^n \cdot x < 8x - 1$ nema rješenja kad je

$n=3$

D

$2^3 \cdot x - 8x < -1$

$0 < -1$

DZ zadaci - 2, 4, 7, 12 ispit 4 - 5, 6, 9
ispit 1 - 3, 5, 7, 9, 10
ispit 2 - 5, 7, 10
ispit 3 - 1, 8, 9

6. SKUP KOMPLEKSNIH BROJEVA

(1) (Primjer 3)

Izračunajmo:

$$\left(\frac{i^{8n-5}+i^{4n+6}}{i^{12n-8}-i^{16n+9}}\right)^{44n+55}=(\circledast)$$

$$i^{8n-5}=i^{8n-8+3}=\underbrace{i^{8n-8}}_{=1,}\cdot\underbrace{i^{3}}_{=-i}=-i$$

jer je eksponent imaginarne jedinice djeljiv s 4

$$i^{4n+6}=\underbrace{i^{4n+4}}_{=1}\cdot\underbrace{i^{2}}_{=-1}=1\cdot(-1)=-1$$

obostrano eksponenta djeljiva s 4

$$i^{12n-8}=1$$

$$i^{16n+9}=\underbrace{i^{16n+8}}_{=1}\cdot i=i$$

$$(\circledast)=\left(\frac{-i-1}{1-i}\right)^{44n+55}=\left(\frac{-(1+i)}{1-i}\right)^{44n+55}=\left(\frac{-(1+i)(1+i)}{(1-i)(1+i)}\right)^{44n+}$$

množimo brojnik i nazivnik s $(1+i)$

$$= \left(\frac{-(1+i)^2}{1-i^2} \right)^{44n+35} = \left(\frac{-(\cancel{1}+2i-\cancel{1})}{1-(-1)} \right)^{44n+35} =$$

$$= \left(\frac{-\cancel{2}i}{\cancel{2}} \right)^{44n+35} = (-i)^{44n+35} = (-i)^{44n-3} \cdot \underbrace{(i)^{52}}_{=1} =$$

$$= (-1) \cdot (-i) \cdot 1 = \boxed{i}$$

(2) Ako je $z = \frac{1+i\sqrt{3}}{1-i}$,
(Pr 5)
koliki je zbroj $\operatorname{Re} z + \operatorname{Im} z$?

$$z = \underbrace{\operatorname{Re} z}_{x} + \underbrace{\operatorname{Im} z}_{y}\, i$$

raspišimo z:

$$z = \frac{1+i\sqrt{3}}{1-i} = \frac{(1+i\sqrt{3}) \cdot (1+i)}{(1-i) \cdot (1+i)} =$$

$$= \frac{1-\sqrt{3}+i\sqrt{3}+i}{1-i^2} = \frac{1-\sqrt{3}+i(\sqrt{3}+1)}{2}$$

$$\boxed{\operatorname{Re} z + \operatorname{Im} z = \frac{1-\cancel{\sqrt{3}}}{2} + \frac{1+\cancel{\sqrt{3}}}{2} = \frac{1}{2} + \frac{1}{2} = 1}$$

③ Odredimo realne brojeve a i b iz jednakosti:

(Pr 6) $$\frac{a+bi}{1-i} = \frac{5}{1-2i} \quad / \cdot (1-i)\cdot(1-2i)$$

$$(a+bi)(1-2i) = 5(1-i)$$

$$a + bi - 2ai + 2b = 5 - 5i$$

$$a + 2b + (-2a+b)i = 5 - 5i$$

$$\left.\begin{aligned} a + 2b &= 5 \\ -2a + b &= -5 \quad / \cdot (2) \end{aligned}\right\} (1)$$

$$\left.\begin{aligned} a + \cancel{2b} &= 5 \\ 4a - \cancel{2b} &= 10 \end{aligned}\right\} (1)$$

$$5a = 15 \Rightarrow \boxed{a = 3}$$

$$\Rightarrow 2b = 5 - a$$

$$\boxed{b = \frac{5-a}{2} = \frac{5-3}{2} = 1}$$

④ (Primjer 8)

Izračunajmo: $$\left| \frac{(1-2i)^5}{(2-i)^7 \cdot (1-i)^4} \right| = (\circ)$$

$$(\circ) = \frac{|(1-2i)^5|}{|(2-i)^7 \cdot (1-i)^4|} = \frac{|(1-2i)|^5}{|(2-i)|^7 \cdot |(1-i)|^4} = (\circ\circ)$$

$$|1-2i| = \sqrt{1^2 + (-2)^2} = \sqrt{5}$$

$$|2-i| = \sqrt{2^2 + (-1)^2} = \sqrt{5}$$

$$|1-i| = \sqrt{1^2 + (-1)^2} = \sqrt{2}$$

$$(\circ\circ) = \frac{\cancel{(\sqrt{5})^{4}}}{\cancel{(\sqrt{5})^{4}}^{2}\cdot(\sqrt{2})^{4}} = \frac{1}{5\cdot 4} = \boxed{\frac{1}{20}}$$

5 (Primjer 9)

Odredimo sve kompleksne brojeve za koje je

$$z^2+\overline{z}^2=6$$

$$z\cdot\overline{z}=5$$

$$z=a+bi$$

$$\overline{z}=a-bi$$

$$(a+bi)^2+(a-bi)^2=6$$

$$a^2+\cancel{2abi}-b^2+a^2-\cancel{2abi}-b^2=6$$

$$2a^2-2b^2=6 \quad /:2$$

$$a^2-b^2=3$$

$$(a+bi)(a-bi)=5$$

$$a^2-b^2\cdot(-1)=5$$

$$a^2+b^2=5$$

$(+)$

$$2a^2=5+3=8$$

$$a^2=2$$

$$b^2=1$$

$$\boxed{\begin{aligned} z_{1,2}&=\pm(2+i)\\ z_{3,4}&=\pm(2-i)\end{aligned}}$$

(6) (Primjer 12)

Odredimo skup svih točaka z kompleksne ravnine koji je određen uvjetom $|z-2|^2 + |z+2|^2 = 26$.

$$z = x + yi$$

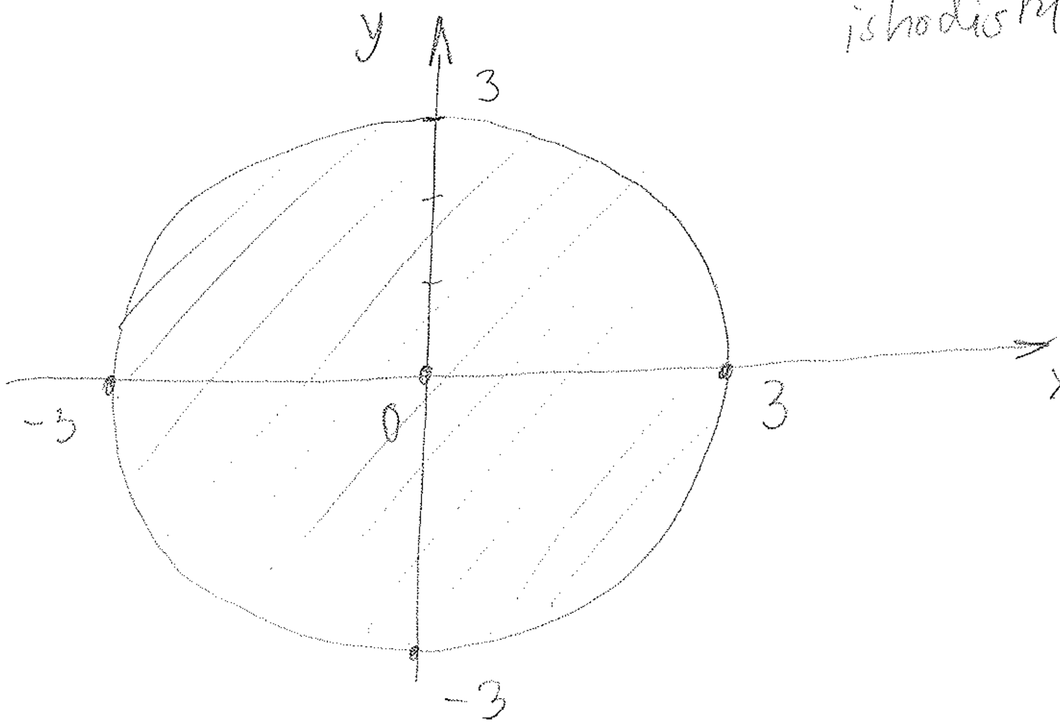
$$|x-2+yi|^2 + |x+2+yi|^2 = 26$$

$$(x-2)^2 + y^2 + (x+2)^2 + y^2 = 26$$

$$x^2 - \cancel{4x} + 4 + y^2 + x^2 + \cancel{4x} + 4 + y^2 = 26$$

$$2x^2 + 2y^2 = 26 - 4 - 4 = 18 \quad /:2$$

$$\underline{x^2 + y^2 = 9} \rightarrow \text{jednadžba kružnice sa središtem u ishodištu}$$



7 $(1+ai)(1-2i) = (1+i)(1-bi)$

$$1 - 2i + ai - 2a\underbrace{i^2}_{=-1}$$

$$= 1 - bi + i - b\underbrace{i^2}_{-1}$$

$1 + 2a + (a-2)i$ A

$$1 - 2i + ai + 2a =$$

$$= 1 + b + (1-b)i$$

$$\left.\begin{aligned} 1 + 2a &= 1 + b \\ a - 2 &= 1 - b \end{aligned}\right\} (+)$$

$$-1 + 3a = 0$$

$$a = \frac{1}{3}$$

$$\frac{1}{3} - 2 = 1 - b \rightarrow b = 1 + 2 - \frac{1}{3}$$

$$b = 3 - \frac{1}{3} = \frac{8}{3}$$

$$\boxed{a+b} = \frac{1}{3} + \frac{8}{3} = \frac{9}{3} = \boxed{3}$$

$$(1+ai)(1-2i) = (1+i)(1-bi)$$

$$1 - 2i + ai - 2ai^2 = 1 - bi + i - bi^2$$

4

$$= 1 + 2a + (a-2)i = 1 + b + (1-b)i$$

$$\begin{cases} 1+2a = 1+b \\ a-2 = 1-b \end{cases}$$

$$-1 + 3a = 0$$

$$a = \frac{1}{3}$$

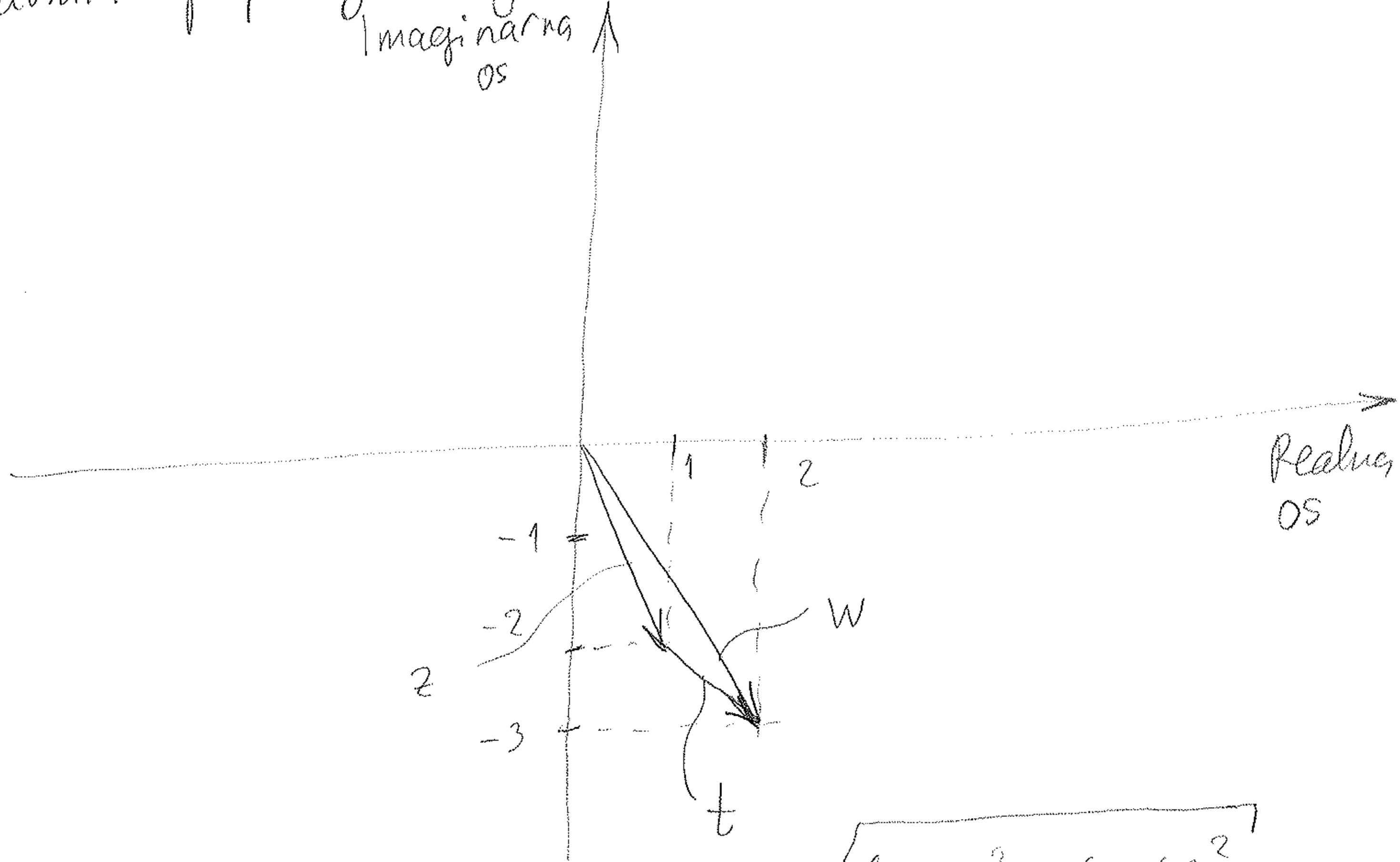
$$\frac{1}{3} - 2 = 1 - b$$

$$b = 1 + 2 - \frac{1}{3}$$

$$b = 3 - \frac{1}{3} = \frac{8}{3}$$

$$\boxed{a+b} = \frac{1}{3} + \frac{8}{3} = \frac{9}{3} = 3$$

⑦ (zadaci - 18)
Kolika je udaljenost točaka koje u kompleksnoj ravnini pripadaju brojevima $z = 1 - 2i$ i $w = 2 - 3i$?



$$t = \sqrt{(2-1)^2 + (-3-(-2))^2}$$

$$t = \sqrt{1 + 1} = \sqrt{2}$$

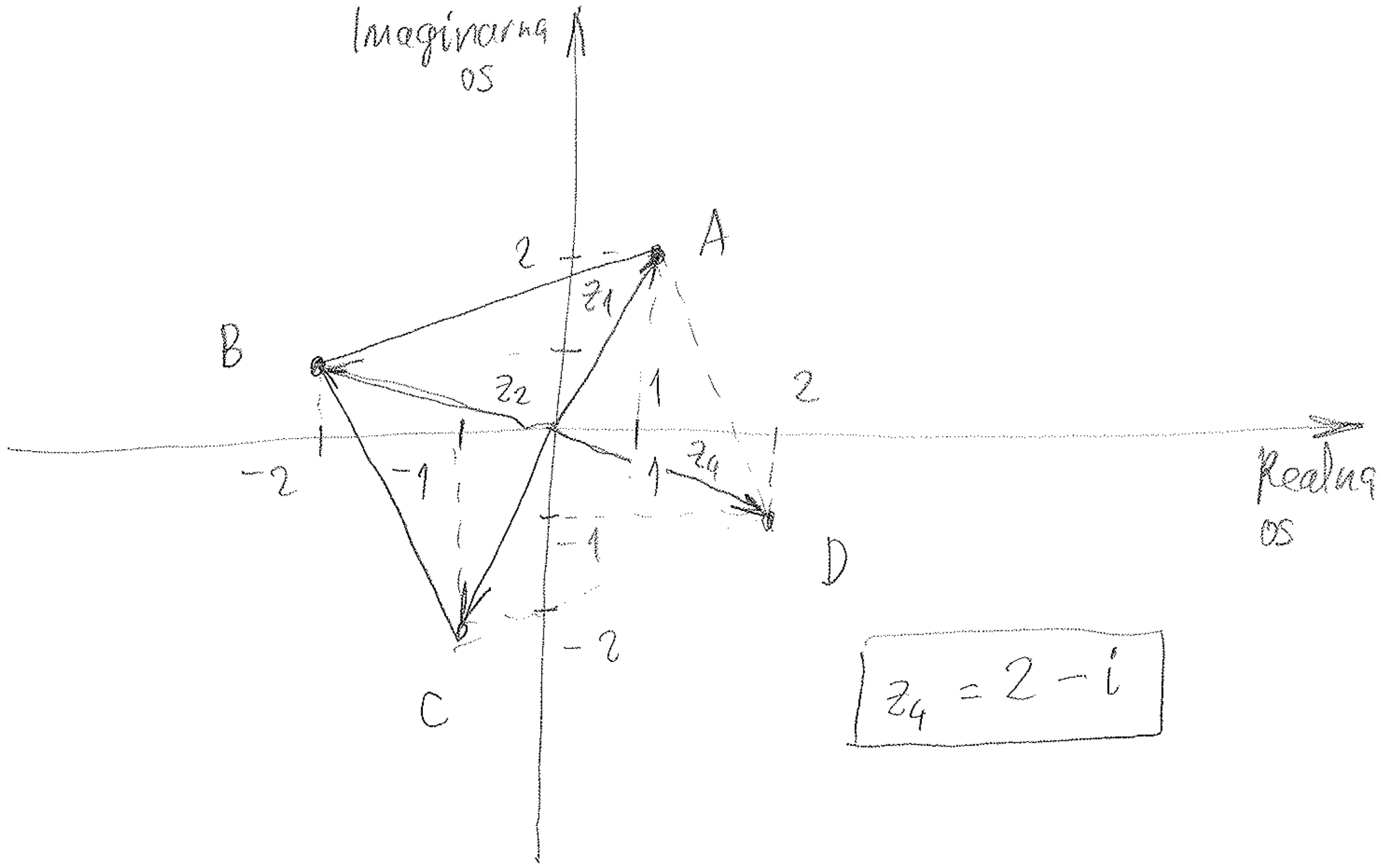
⑧ (zadaci - 19)
Točke kompleksne ravnine koje su pridružene brojevima $z_1 = 1 + 2i$, $z_2 = -2 + i$, $z_3 = -1 - 2i$ vrhovi su kvadrata. Odredite kompleksni broj pridružen četvrtom vrhu tog kvadrata.

$z_1 = 1 + 2i$ ...... A

$z_2 = -2 + i$ ...... B

$z_3 = -1 - 2i$ C

$z_4 = ?$ ...... D



$\boxed{z_4 = 2 - i}$

DZ

zadaci – 1, 4, 10, 11, 15, 21

ispit 1 – 3, 6, 7, 10

ispit 2 – 3, 6, 8, 9

(Ispit 1)

4 $z=\frac{2-i}{1+i}$ $\quad \text{Im}\,\frac{1}{\bar{z}}$

$$z=\frac{2-i}{1+i}\cdot\frac{1-i}{1-i}=\frac{(2-i)(1-i)}{1-i^2}$$

$$z=\frac{2-2i-i-i^2}{2}=\frac{2-3i-1}{2}$$

$$z=\frac{1-3i}{2}$$

$$\bar{z}=\frac{1+3i}{2}$$

$$\frac{1}{\bar{z}}=\frac{2}{1+3i}\cdot\frac{1-3i}{1-3i}$$

$$=\frac{2-6i}{1-9\cdot(-1)}=\frac{2-6i}{10}=\frac{1-3i}{5}$$

$$\text{Im}\left(\frac{1}{\bar{z}}\right)=-\frac{3}{5}$$

B

(1 bild)

4) $z = \frac{2-i}{1+i}$ $\qquad \operatorname{Im} \frac{1}{\bar{z}}$

$$z = \frac{2-i}{1+i} \cdot \frac{1-i}{1-i} = \frac{(2-i)(1-i)}{1-i^2}$$

$$z = \frac{2-2i-i-i^2}{2} = \frac{2-3i-1}{2}$$

$$z = \frac{1-3i}{2}$$

$$\bar{z} = \frac{1+3i}{2}$$

$$\frac{1}{\bar{z}} = \frac{2}{1+3i} \cdot \frac{1-3i}{1-3i}$$

$$= \frac{2-6i}{1-9\cdot(-1)} = \frac{2-6i}{10} = \frac{1-3i}{5}$$

$$\operatorname{Im}\left(\frac{1}{\bar{z}}\right) = -\frac{3}{5}$$ B

Ispiti 6

6

$$w = \frac{\bar{z}}{z^2}$$

$$z = 1 - 3i$$

$$\bar{z} = 1 + 3i$$

$$z^2 = (1-3i)^2 = 1 - 6i + 9(-1)$$
$$= 1 - 9 - 6i = -8 - 6i$$

$$w = \frac{1+3i}{-8-6i} \cdot \frac{-8+6i}{-8+6i} = \dots$$

$$|w| = \sqrt{(\operatorname{Re} w)^2 + (\operatorname{Im} w)^2}$$

$$\dots = \frac{(1+3i)(-8+6i)}{64 - 6^2 \cdot i^2} = \frac{-8+6i-24i-18}{100}$$

$$= -\frac{26}{100} - \frac{18}{100}i = -\frac{13}{50} - \frac{9}{50}i$$

$$\sqrt{\frac{169+81}{2500}}$$

$$= \sqrt{\frac{250}{2500}}$$

$$= \frac{\cancel{5}^{1}\sqrt{10}}{\cancel{50}_{10}}$$

C

[illegible] 6

(6)

$w = \frac{\bar z}{z^2}$

$z = 1 - 3i$

$\bar z = 1 + 3i$

$z^2 = (1-3i)^2 = 1 - 6i + 9(-1)$

$= 1 - 9 - 6i = -8 - 6i$

$w = \frac{1+3i}{-8-6i} \cdot \frac{-8+6i}{-8+6i} = \dots$

$|w| = \sqrt{(\mathrm{Re}\, w)^2 + (\mathrm{Im}\, w)^2}$ $\qquad -26i - 18$

$\dots = \frac{(1+3i)(-8+6i)}{64 - 6^2 \cdot i^2} = \frac{-26-18i}{100}$

$= -\frac{26}{100} - \frac{18}{100} i = -\frac{13}{50} - \frac{9}{50} i$

(c)

$\sqrt{\frac{169+81}{2500}} = \sqrt{\frac{250}{2500}} = \frac{\sqrt{10}}{10}$

7. KVADRATNA FUNKCIJA, JEDNADŽBE I NEJEDNADŽBE

① (Primjer 2)
Riješimo jednadžbu $\frac{x+1}{2x^2} + \frac{3x^2}{x+1} = \frac{5}{2}$

$$t = \frac{x+1}{x^2}$$

$$\frac{1}{2}t + \frac{3}{t} = \frac{5}{2} \quad / \cdot 2t$$

$$t^2 + 6 = 5t$$

$$t^2 - 5t + 6 = 0$$

$$t_{1,2} = \frac{5 \pm \sqrt{25 - 4 \cdot 6}}{2} \quad \begin{matrix} \nearrow t_1 = 3 \\ \searrow t_2 = 2 \end{matrix}$$

za $t_1 = 3$

$$\frac{x+1}{x^2} = 3$$

$$3x^2 - x - 1 = 0$$

$$x_{1,2} = \frac{1 \pm \sqrt{(-1)^2 - 4 \cdot 3 \cdot (-1)}}{2 \cdot 3}$$

$$x_{1,2} = \frac{1 \pm \sqrt{1 + 12}}{6} \quad \begin{matrix} \nearrow \\ \searrow \end{matrix} \boxed{\begin{matrix} x_1 = \frac{1+\sqrt{13}}{6} \\ x_2 = \frac{1-\sqrt{13}}{6} \end{matrix}}$$

za $t_1 = 2$

$$\frac{x+1}{x^2} = 2$$

$$2x^2 - x - 1 = 0$$

$$x_{3,4} = \frac{1 \pm \sqrt{(-1)^2 - 4 \cdot 2 \cdot (-1)}}{2 \cdot 2}$$

$$x_{3,4} = \frac{1 \pm \sqrt{1 + 8}}{4}$$

$$\boxed{x_3 = 1 \qquad x_4 = -\frac{1}{2}}$$

② (Primjer 3)

Riješimo jednadžbu: $\sqrt{x+1}+\sqrt{x+2}=\sqrt{x+3}$ (1)

$$\left.\begin{array}{l} x+1 \geq 0 \\ x+2 \geq 0 \\ x+3 \geq 0 \end{array}\right\} \Rightarrow x \geq -1$$

za realna rješenja naravno ako ih uopće ima!

$(1)/^2$

$$\left(\sqrt{x+1}+\sqrt{x+2}\right)^2=\left(\sqrt{x+3}\right)^2$$

$$x+1+2\sqrt{(x+1)(x+2)}+x+2=x+3$$

$$2x+2\sqrt{(x+1)(x+2)}+\cancel{3}=x+\cancel{3}$$

$$x+2\sqrt{(x+1)(x+2)}=0$$

$$2\sqrt{(x+1)(x+2)}=-x \;/^2$$

$$4(x+1)(x+2)=x^2$$

$$4(x^2+2x+x+2)=x^2$$

$$4(x^2+3x+2)=x^2$$

$$3x^2+12x+8=0$$

$$x_{1,2}=\frac{-12\pm\sqrt{144-4\cdot 3\cdot 8}}{2\cdot 3}$$

$$x_{1,2}=\frac{-12\pm\sqrt{144-96}}{6}$$

$$\boxed{x_{1,2} = -2 \pm \frac{\cancel{4}^{2}\sqrt{3}}{\cancel{6}_{3}} = -2 \pm \frac{2\sqrt{3}}{3}}$$

③ (Primjer 5)
Zbroj rješenja jednadžbe $x^2+mx+n=0$ jednak je -2, a razlika rješenja iznosi 5. Odredimo brojeve m i n.

$$x^2+mx+n=0$$

$$x_{1,2} = \frac{-m \pm \sqrt{m^2-4n}}{2} \begin{cases} \nearrow x_1 = \frac{-m+\sqrt{m^2-4n}}{2} \\ \searrow x_2 = \frac{-m-\sqrt{m^2-4n}}{2} \end{cases}$$

$$x_1+x_2 = \frac{-m+\cancel{\sqrt{m^2-4n}}}{2} + \left(\frac{-m-\cancel{\sqrt{m^2-4n}}}{2}\right)$$

$$x_1+x_2 = -m = -2 \Rightarrow m=2$$

$$x_1-x_2 = 5$$

$$\sqrt{m^2-4n} = 5 \quad /^2$$

$$m^2-4n = 25$$

$$4n = m^2-25 = 4-25 = -21$$

$$\boxed{n = -\frac{21}{4}}$$

④ (Primjer 7)
Riješimo sustav jednadžbi $\begin{cases} x^2y+xy^2 = 20 \\ \frac{1}{x}+\frac{1}{y} = \frac{5}{4} \end{cases}$

→ simetrični sustav jednadžbi

$$\left.\begin{aligned} xy(x+y) &= 20 \\ \frac{x+y}{xy} &= \frac{5}{4} \end{aligned}\right\} (\cdot) \Rightarrow \cancel{xy}^{1}(x+y)\frac{(x+y)}{\cancel{xy}_{1}} = \cancel{20}^{5}\cdot\frac{5}{\cancel{4}_{1}} = 2\ldots$$

$$(x+y)^2 = 25$$

može biti nakon $\sqrt{\ }$

$$\left.\begin{array}{l} \text{I)}\ x+y=5 \\ \text{II)}\ x+y=-5 \end{array}\right\} \Rightarrow \begin{array}{l} xy=4 \\ xy=-4 \end{array}$$

$$\text{I)}\ \left.\begin{array}{l} x+y=5 \\ xy=4 \Rightarrow y=\frac{4}{x} \end{array}\right\} \quad x+\frac{4}{x}=5$$

$$\frac{x^2+4}{x}=5$$

$$x^2-5x+4=0$$

$$x_{1,2}=\frac{5\pm\sqrt{25-16}}{2}$$

$$x_{1,2}=\frac{5\pm 3}{2} \quad \boxed{\begin{array}{l} x_1=4 \\ x_2=1 \end{array}}$$

$$\text{II)}\ \left.\begin{array}{l} x+y=-5 \\ xy=-4 \end{array}\right\} \quad x-\frac{4}{x}=-5$$

$$\Downarrow$$

$$y=-\frac{4}{x}$$

$$\frac{x^2-4}{x}=-5$$

$$x^2+5x-4=0$$

$$x_{3,4}=\frac{-5\pm\sqrt{25+16}}{2}$$

$$\boxed{\begin{array}{l} x_3=\dfrac{-5+\sqrt{41}}{2} \\ x_4=\dfrac{-5-\sqrt{41}}{2} \end{array}}$$

⑤ (Primjer 12)

Dana je funkcija $f(x) = -x^2 + 2x + 3$.

1) Za koju vrijednost od x ova funkcija prima ekstremnu vrijednost?
2) Odredimo skup svih vrijednosti polinoma f.
3) Odredimo nultočke funkcije f.
4) Nacrtajmo graf funkcije i opišimo njezin tok.

1) Ekstremna vrijednost funkcije

$$\boxed{x_0 = -\frac{b}{2a} = \frac{-2}{2\cdot(-1)} = 1}$$

$$f(x_0 = -1) = -1^2 + 2\cdot 1 + 3 = -1 + 2 + 3 = 4$$

2) $f(x) \in \langle -\infty, 4]$

3) $-x^2 + 2x + 3 = 0$

↳ nultočke funkcije

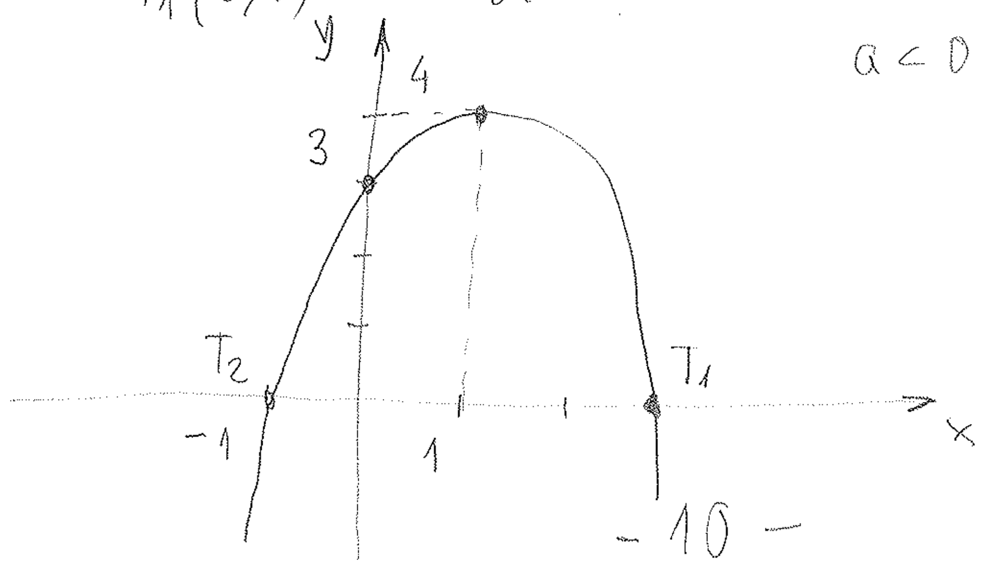
$$x_{1,2} = \frac{-2 \pm \sqrt{4+12}}{-2} = \frac{-2 \pm 4}{-2} \begin{cases} \nearrow x_1 = 3 \\ \searrow x_2 = -1 \end{cases}$$

$$f(x_1 = 3) = -3^2 + 2\cdot 3 + 3 = 0$$

$$f(x_2 = -1) = -(-1)^2 + 2\cdot(-1) + 3 = 0$$

$T_1(3, 0)$ $T_2(-1, 0)$

4)



$a < 0 \rightarrow f(x)$ okrenuta prema dolje

⑥ (Primjer 18)
Odredimo jednadžbu tangente na parabolu $y = -x^2 + x + 1$ ako je ta tangenta paralelna pravcu $y = 3x + 11$. Odredimo i koordinate dirališta pravca i parabole.

$y_1 \ldots y = -x^2 + x + 1 \ldots$ parabola

$$x_{1,2} = \frac{-1 \pm \sqrt{1 - 4 \cdot (-1) \cdot 1}}{-2} = \frac{1}{2} \pm \frac{\sqrt{5}}{2}$$

$$x_1 = \frac{1 - \sqrt{5}}{2} = -0,62$$

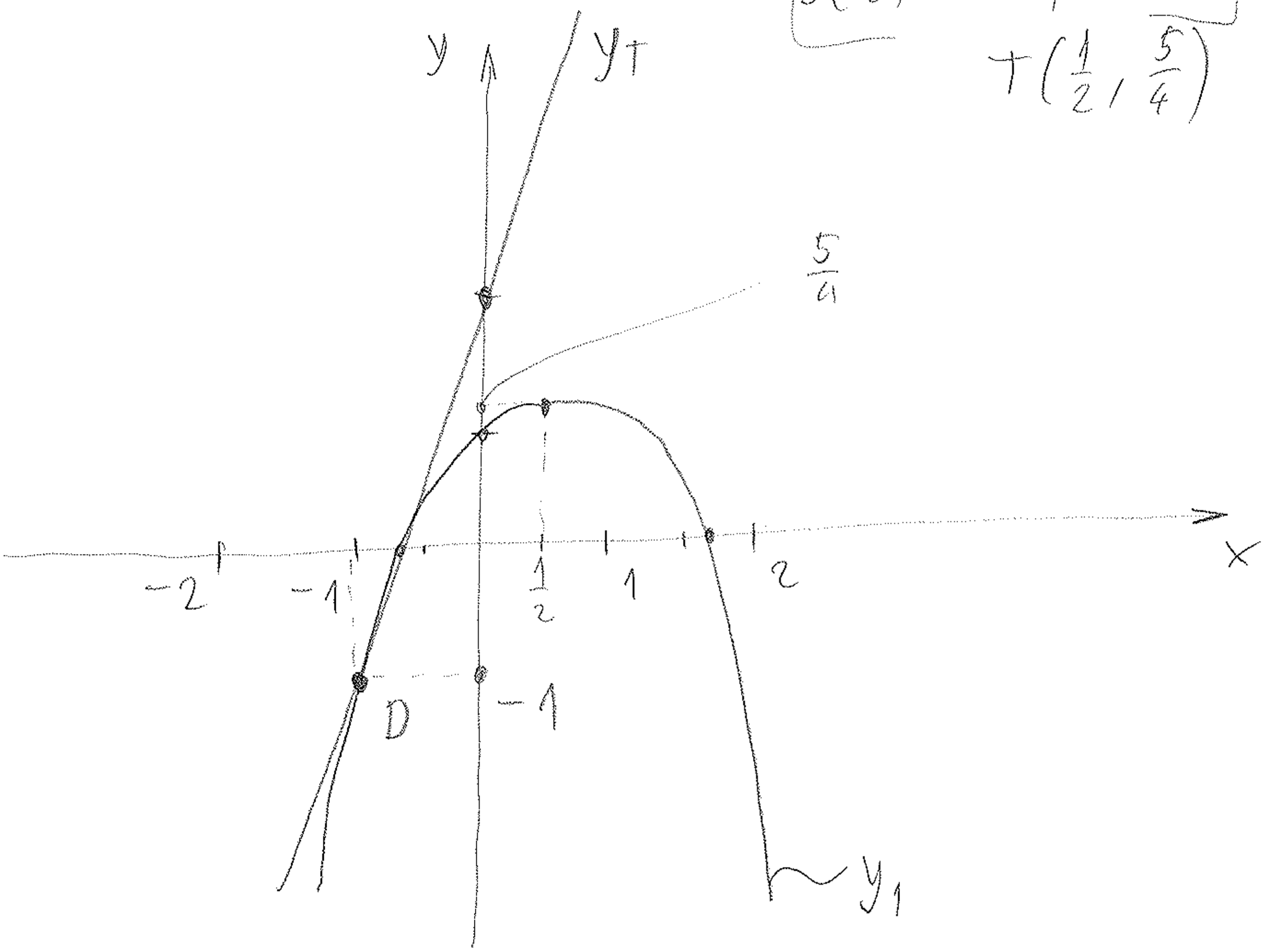
$$x_2 = \frac{1 + \sqrt{5}}{2} = 1,62$$

tjeme parabole $\boxed{x_0 = \frac{-b}{2a} = \frac{-1}{-2} = \frac{1}{2}}$

$$y\left(\frac{1}{2}\right) = -\frac{1}{4} + \frac{1}{2} + 1 = \frac{3}{4} + \frac{1}{2}$$

$$\boxed{y\left(\frac{1}{2}\right) = \frac{3+2}{4} = \frac{5}{4}}$$

$$T\left(\frac{1}{2}, \frac{5}{4}\right)$$



$\underbrace{t \parallel y_2}$ ... $y = 3x + 11$

$\Rightarrow a_T = a_2 = 3$

$y_T = 3x + b_T$?

$D > 0$ pravac i parabola se sijeku u dvije različite točke

$D < 0$ pravac i parabola uopće se ne sijeku

$D = 0$ pravac i parabola se sijeku u jednoj točki ✓

$D = b^2 - 4ac$

$\hookrightarrow$ kvadratna jednadžba (sustav)

$y_1 = -x^2 + x + 1$

$y_T = 3x + b_T$

$-x^2 + x + 1 = 3x + b_T$

$-x^2 - 2x - b_T + 1 = 0 \quad / \cdot (-1)$

$x^2 + 2x + b_T - 1 = 0$

$D = 4 - 4 \cdot 1 \cdot (b_T - 1) = 0$

$4 - 4b_T + 4 = 0 \Rightarrow b_T = 2$

$\boxed{y_T = 3x + 2}$ jednadžba tangente

Koordinate dirališta:

$$x^2 + 2x + 1 = 0$$

$$(x+1)^2 = 0 \implies x_1 = -1$$
$$x_2 = 1$$

$$y_T(x=-1) = 3\cdot(-1) + 2 = -3+2 = -1$$

$$\boxed{D(-1,-1)}$$

(7) (Zadaci - 26)

Riješite nejednadžbe:

1) $$\frac{2x-1}{x^2-x+1} < 1$$

$$\frac{2x-1}{x^2-x+1} - 1 < 0$$

$$\frac{2x-1-(x^2-x+1)}{x^2-x+1} < 0$$

$$\frac{2x-1-x^2+x-1}{x^2-x+1} < 0$$

$y_1 \ldots$ $y_2 \ldots$ $$\frac{-x^2+3x-2}{x^2-x+1} < 0$$

dvije parabole

$$y_1 \ldots -x^2+3x-2 = 0$$

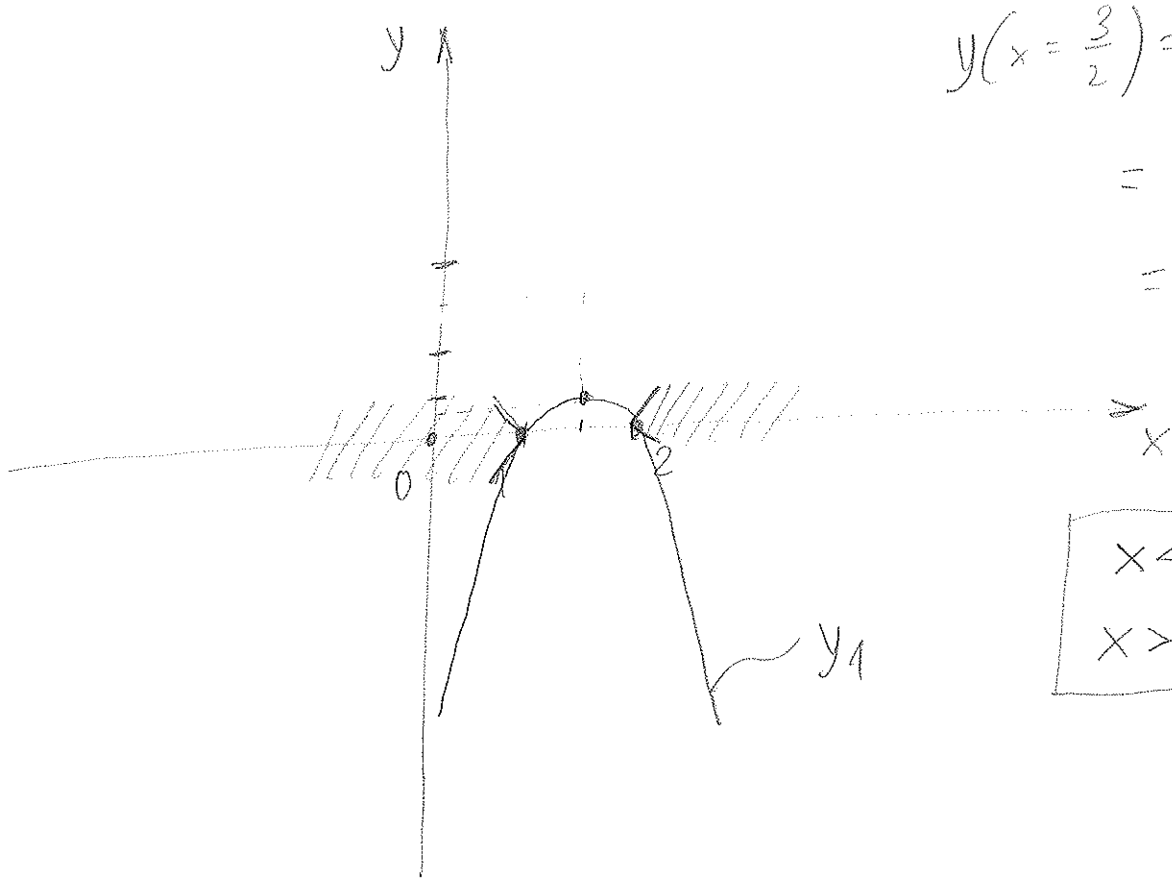
$$x_{1,2} = \frac{-3 \pm \sqrt{9-4\cdot(-2)\cdot}}{(-1)\cdot 2}$$

$$x_{1,2} = \frac{-3 \pm \sqrt{9-8}}{-2}$$

$$x_{1,2} = \frac{3}{2} \pm \frac{1}{2} \nearrow x_1 = \quad \searrow x_2 =$$

tjeme: $x_0 = -\frac{b}{2a} = \frac{-3}{-2} =$

$$y\left(x=\frac{3}{2}\right)=-\left(\frac{3}{2}\right)^2+3\cdot\frac{3}{2}-2$$
$$=-\frac{9}{4}+\frac{9}{2}-2$$
$$=\frac{-9+18-8}{4}$$



$$\boxed{\begin{array}{l} x<1 \\ x>2 \end{array}}$$

$y_2 \cdots x^2-x+1=0$

$$x_{1,2}=\frac{1\pm\sqrt{1-4}}{2}$$
$$x_{1,2}=\frac{1\pm\sqrt{-3}}{2}=\frac{1\pm\sqrt{3}\,i}{2}$$

$D<0$

nema nul točak

2) $$\frac{x-1}{x^2+3x-4}\geq 1$$

$$\frac{x-1}{x^2+3x-4}-1\geq 0$$

$$\frac{x-1-(x^2+3x-4)}{x^2+3x-4}\geq 0$$

$$\begin{array}{l} P_1 \cdots \\ P_2 \cdots \end{array}\frac{-x^2-2x+3}{x^2+3x-4}\geq 0$$

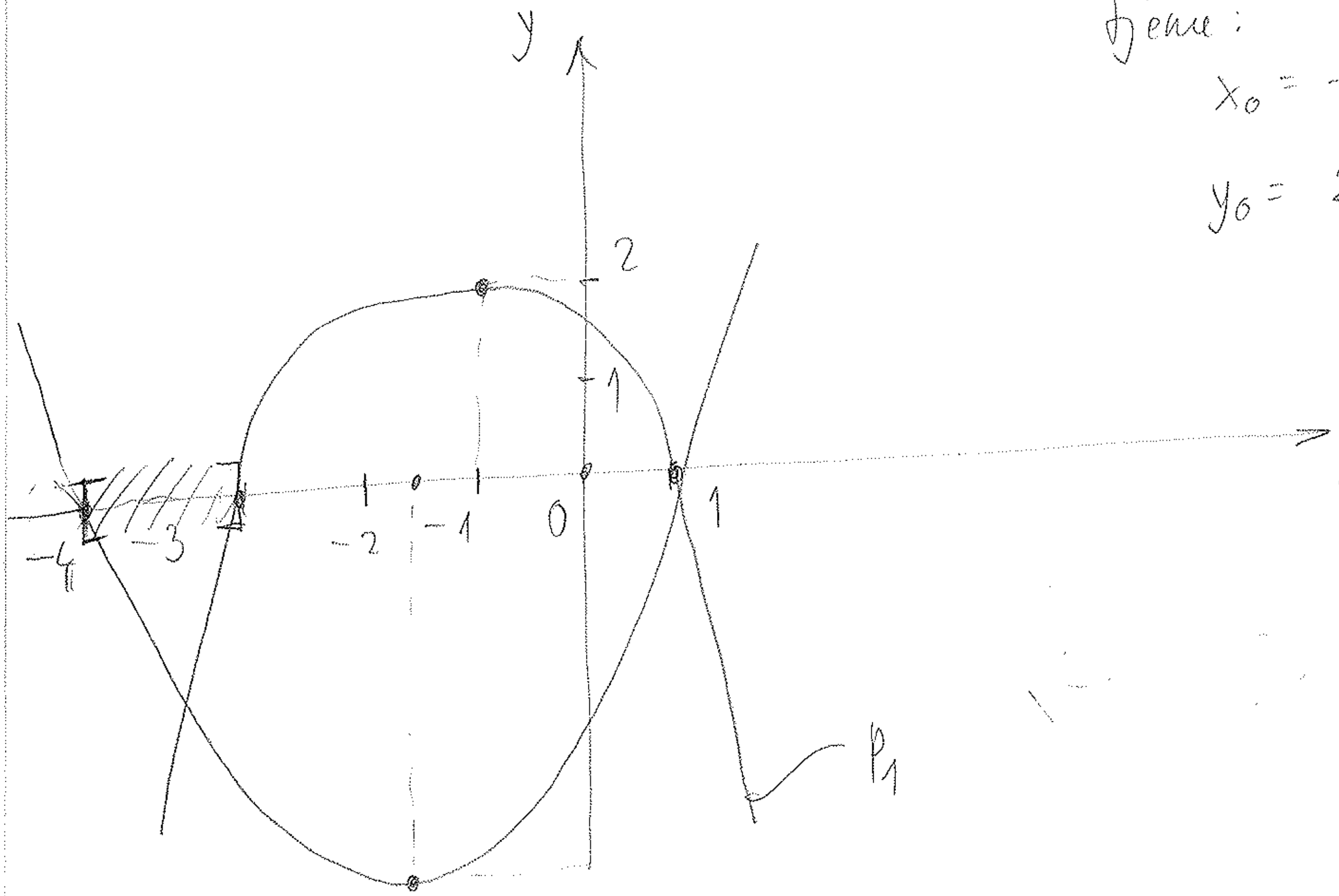
P_1 ... $-x^2-2x+3=0$

$$x_{1,2}=\frac{2\pm\sqrt{4+12}}{-2}=\frac{2\pm 4}{-2} \quad \nearrow x_1=-3, \quad \searrow x_2=1$$

tjeme:

$$x_0=-\frac{b}{2a}=\frac{2}{-2}=$$

$$y_0=2$$



$x^2+3x-4=0$

$$x_{1,2}=\frac{-3\pm\sqrt{9+16}}{2}=\frac{-3\pm 5}{2} \quad \nearrow x_1=1, \quad \searrow x_2=-4$$

tjeme: $x_0=-\frac{b}{2a}=-\frac{3}{2}$

$$y_0=\frac{9}{4}-\frac{9}{2}-4$$

$$y_0=\frac{9-18-16}{4}$$

$$y_0=-\frac{25}{4}\approx -6,25$$

Da bi odnos parabola bio ≥ 0 obje moraju biti ili obje pozitivne ili negativne na intervalu!

$[-4,-3]$ obje negativne

8 (6 - ispit 1)

Sustav jednadžbi $x+y=a$ i $xy=4$ nema realnih rješenja ako je:

$\hookrightarrow x = \frac{4}{y}$

nema realnih rješenja $D<0$

$\hookrightarrow$ diskriminanta suokrom

$$x+y=a$$

$$\frac{4}{y}+y=a \quad / \cdot y$$

$$4+y^2=ay$$

$$y^2-ay+4=0$$

$$D=b^2-4ac<0$$

$$a^2-4\cdot 4<0$$

$$a^2<16 \quad / \sqrt{\ }$$

$$\left.\begin{array}{l} a<4 \\ a>-4 \end{array}\right\} \quad C$$

9 Tjeme parabole je na osi x. Parabola $y=-2x^2+3x+c$. Koliki je c?

$\rightarrow D=0$

$$D=b^2-4ac=0$$

$$9+4\cdot 2\cdot c \Rightarrow \boxed{c=-\frac{9}{8}} \quad D$$

(10) (ispit 3 - 2) (-)

Umnožak svih rješenja jednadžbe $\sqrt{\frac{x-1}{x+2}}+\sqrt{\frac{x+2}{x-1}}=\frac{5}{2}$ jednak je:

$2-\frac{25}{4}=$

$$\left(\frac{x-1}{x+2}\right)+\frac{x+2}{x-1}+2=\frac{25}{4} \quad / \cdot (x+2)(x-1)$$

$$(x-1)^2+(x+2)^2-\frac{17}{4}(x+2)(x-1)=0$$

$$(x-1)^2+(x+2)^2-\frac{17}{4}(x^2-x+2x-2)=0$$

$$x^2-2x+1+x^2+4x+4-\frac{17}{4}x^2+\cancel{\frac{17}{4}x}-\frac{17}{2}x+\frac{17}{2}=0$$

$$2x^2+2x+5-\frac{17}{4}x^2$$

$$2x^2+2x+5-\frac{17}{4}x^2-\frac{17}{4}x+\frac{17}{2}=0$$

$$-\frac{9}{4}x^2-\frac{9}{4}x+\frac{27}{2}=0 \quad / \cdot -\frac{4}{9}$$

$$\cancel{\frac{\frac{9}{4}\pm\sqrt{\frac{81}{16}+4\cdot\frac{9}{4}\cdot\frac{27}{2}}}{2\cdot\left(-\frac{9}{4}\right)}}$$

$$\cancel{x_{1,2}=\frac{\frac{9}{4}\pm\sqrt{\frac{81}{16}+\frac{243}{2}}}{-\frac{9}{2}}}$$

$$\cancel{x_{1,2}=\frac{9}{4}\cdot\left(-\frac{1}{9}\right)\pm\frac{\sqrt{\frac{81+1944}{16}}}{-\frac{9}{2}}}$$

$$-x^2-x+6=0$$

$$x^2+x-6=0$$

$$x_{1,2}=\frac{-1\pm\sqrt{1+6\cdot 4}}{2}$$

$$x_{1,2}=\frac{-1\pm 5}{2} \quad x_1=-3, \quad x_2=2$$

(C)

$$\boxed{x_1\cdot x_2=-3\cdot 2=\boxed{-6}}$$

$$x_{1,2} = -\frac{1}{2} \pm \frac{\cancel{45}^{5}}{\cancel{4}_{1}} \cdot \left(-\frac{\cancel{2}^{1}}{\cancel{9}_{1}}\right) = -\frac{1}{2} \pm 5$$

$$\left.\begin{aligned} x_1 &= \frac{11}{2} \\ x_2 &= -2 \end{aligned}\right\} \quad x_1 \cdot x_2 = \frac{11}{2} \cdot (-2) = -11$$

$-$

<u>DZ</u>

zadaci – 5, 7, 11, 15, 22

ispit 1 – 3, 8, 10

ispit 2 – 5, 9, 10

ispit 3 – 2, 7, 9

ispit 4 – 3, 4, 6, 9, 10

8. EKSPONENCIJALNE I LOGARITAMSKE FUNKCIJE

① Odredimo nepoznanicu x u svakoj od tri sljedeće
(Pr 4) jednakosti:

1) $\log_{\frac{1}{2}} x = -\frac{1}{2}$

$$\left[\log_a b = r \iff a^r = b \right.$$

$$\boxed{x = \left(\frac{1}{2}\right)^{-\frac{1}{2}} = \sqrt{2}}$$

2) $\log_x 16 = \frac{4}{3}$

$$x^{\frac{4}{3}} = 16 \quad \Big/^{\frac{3}{4}}$$

$$\boxed{x = 16^{\frac{3}{4}} = \left(2^{\cancel{4}^{1}}\right)^{\frac{3}{\cancel{4}_{1}}} = 8}$$

3) $\log_8 \sqrt[3]{16} = x$

$$2^{3x} = 8^x = \sqrt[3]{16} = 2^{\frac{4}{3}}$$

$$3x = \frac{4}{3} \quad \Big/ : 3$$

$$\boxed{x = \frac{4}{9}}$$

② Izračunajmo:

(Pr 5) 1) $\boxed{4^{-\log_2 7}} = 2^{2(-\log_2 7)} = (\underbrace{2^{\log_2 7}}_{7})^{-2} = \frac{1}{49}$

2) $\boxed{5^{2+\log_{25} 9}} = 5^2 \cdot \underbrace{5}_{\sqrt{25} = 25^{\frac{1}{2}}}^{\log_{25} 9} = 25 \cdot 25^{\frac{1}{2}\log_{25} 9} =$

$= 25 \cdot 9^{\frac{1}{2}} = 25 \cdot 3 = \boxed{75}$

3) $\boxed{\left(\frac{1}{3}\right)^{-\log_{\sqrt{3}} 12 + 3\log_9 4}} =$

$= \left(\frac{1}{3}\right)^{-\log_{\sqrt{3}} 12} \cdot \left(\frac{1}{3}\right)^{3\log_9 4} = \sqrt{3}^{-(-2)\log_{\sqrt{3}} 12} \cdot$

$\cdot 3^{(-1)\cdot 3\log_9 4} = (\sqrt{3})^{2\log_{\sqrt{3}} 12} \cdot 3^{-3\log_9 4} =$

$= \underbrace{\left(\sqrt{3}^{\log_{\sqrt{3}} 12}\right)^2}_{12^2} \cdot \underbrace{9^{-\frac{3}{2}\log_9 4}}_{4^{-\frac{3}{2}}} = 144 \cdot 2^{-3} = \frac{144}{8} = 18$

③ Ako je $\log_3 2 = m$, koliko je $\log_{18} 36$?

(Pr 8)

$$\log_{18} 36 = \frac{\log_3 36}{\log_3 18} = (\circ)$$

$$\left[\ \log_a x = \frac{\log_b x}{\log_b a}\right.$$

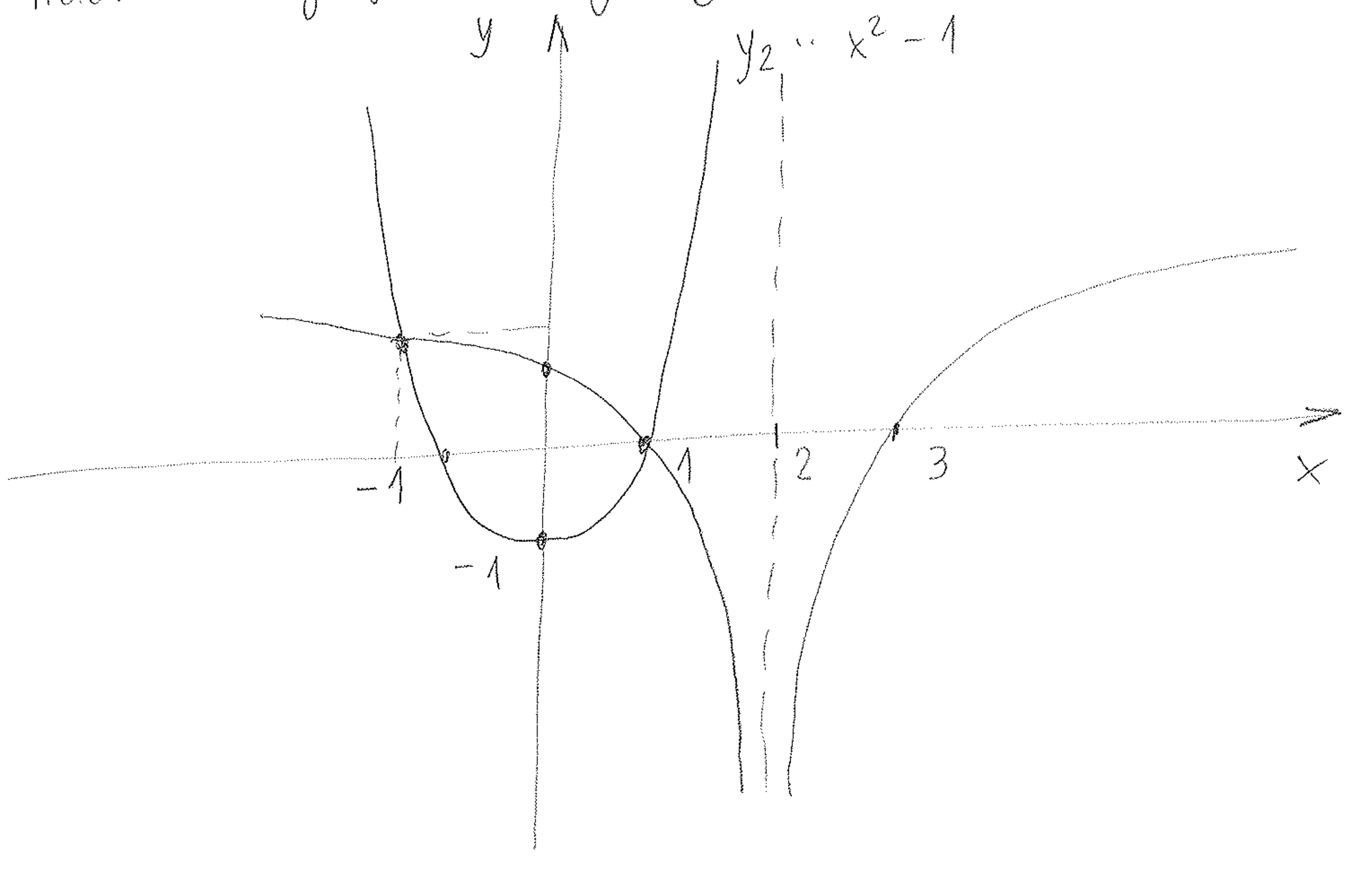
$$(\circ) = \frac{\log_3 9 + \log_3 4}{\log_3 9 + \log_3 2} = \frac{2 + \log_3 2^2}{2 + \log_3 2} = \frac{2(1 + \overbrace{\log_3 2}^{m})}{2 + \underbrace{\log_3 2}_{m}}$$

$$= \frac{2 + 2m}{2 + m} = \boxed{\frac{2(1+m)}{2+m}}$$

④ (Primjer 11)

Koliko rješenja ima jednadžba $\log_2 |x-2| = x^2 - 1$

– nacrtamo grafove na jednoj slici:



$y_1 = \log_2 |x-2|$

I) $y_1 = \log_2(x-2)$, za $x > 2$

II) $y_2 = \log_2(-(x-2)) = \log_2(2-x)$, za $x < 2$

ima 2 rješenja

5 (Primjer 12)
Riješimo jednadžbu

$$27^{2-x} \cdot \sqrt[3]{9^{2x+1}} = \left(\frac{1}{3}\right)^{x-4}$$

$$3^{3(2-x)} \cdot \left(3^{2}\right)^{(2x+1)\cdot\frac{1}{3}} = \left(3^{-1}\right)^{x-4}$$

$$3^{6-3x+\frac{4x+2}{3}} = 3^{4-x}$$

$$3^{\frac{18-9x+4x+2}{3}} = 3^{4-x}$$

$$\Rightarrow \frac{20-5x}{3} = 4-x \quad / \cdot 3$$

$$20-5x = 12-3x$$

$$2x = 8 \Rightarrow \boxed{x=4}$$

6 Riješimo jednadžbu:
(Pr 14)

$$5^x + 5 = 6 \cdot (\sqrt{5})^x$$

$$5^x = (\sqrt{5})^{2x}$$

$$(\sqrt{5})^{2x} - 6 \cdot (\sqrt{5})^x + 5 = 0$$

supstitucija: $t = (\sqrt{5})^x$

$$t^2 - 6 \cdot t + 5 = 0$$

$$t_{1,2} = \frac{6 \pm \sqrt{36 - 20}}{2} = \frac{6 \pm 4}{2} \nearrow t_1 = 5, \quad \searrow t_2 = 1$$

$$(\sqrt{5})^x = 5 = (\sqrt{5})^2 \Rightarrow \boxed{x = 2}$$

$$(\sqrt{5})^x = 1 = 5^0 = (\sqrt{5})^0 \Rightarrow \boxed{x = 0}$$

7 (Primjer 15)

Riješimo jednadžbu $\log(0.1 x^2) \cdot \log\left(\frac{x}{10}\right) = 3$

$$(\log(0.1) + 2 \log x) \cdot (\log x - \log 10) = 3$$

$$(-1 + 2 \log x) \cdot (\log x - 1) = 3$$

$$-\log x + 2 \log^2 x + 1 - 2 \log x = 3$$

$$2 \log^2 x - 3 \log x - 2 = 0$$

Supstitucija: $\log x = t$

$$2t^2 - 3t - 2 = 0$$

$$t_{1,2} = \frac{3 \pm \sqrt{9 + 16}}{4}$$

$$t_{1,2} = \frac{3 \pm 5}{4} \nearrow t_1 = -\frac{1}{2}, \quad \searrow t_2 = 2$$

$$\log x = -\frac{1}{2} \Rightarrow \boxed{x_1 = \frac{\sqrt{10}}{10}}$$

$$\log x = 2 \Rightarrow \boxed{x_2 = 100}$$

(8) (Primjer 17)
Riješimo sustav jednadžbi:

$$\begin{cases} 3^{x-2} \cdot 2^{y-4} = 144 & (1) \\ 1 + \log_2 x = \log_2 y & (2) \end{cases}$$

iz (2): $\Rightarrow$ $\log_2 x - \log_2 y = -1$

$$\log_2 \frac{y}{x} = 1 \Rightarrow 2^1 = \frac{y}{x}$$

$$y = 2x \rightarrow (1)$$

$$3^{x-2} \cdot 2^{2x-4} = 144$$

$$3^{x-2} \cdot 4^{x-2} = 12^{x-2}$$

$$12^{x-2} = 144 = 12^2 \Rightarrow x - 2 = 2$$

$$\boxed{x = 4}$$

$$\boxed{y = 8}$$

(9) (Primjer 18)
Riješimo nejednadžbu: $0{,}8^{2x-3} \leq \left(\frac{\sqrt{5}}{2}\right)^{x+1}$

$$\left(\frac{\cancel{8}^{\,4}}{\cancel{10}_{\,5}}\right)^{2x-3} \leq \left(\frac{5^{\frac{1}{2}}}{4^{\frac{1}{2}}}\right)^{x+1} = \left(\frac{4}{5}\right)^{-\frac{1}{2}(x+1)}$$

$$a<1 \Longrightarrow 2x - 3 \geq -\frac{1}{2}(x+1)$$

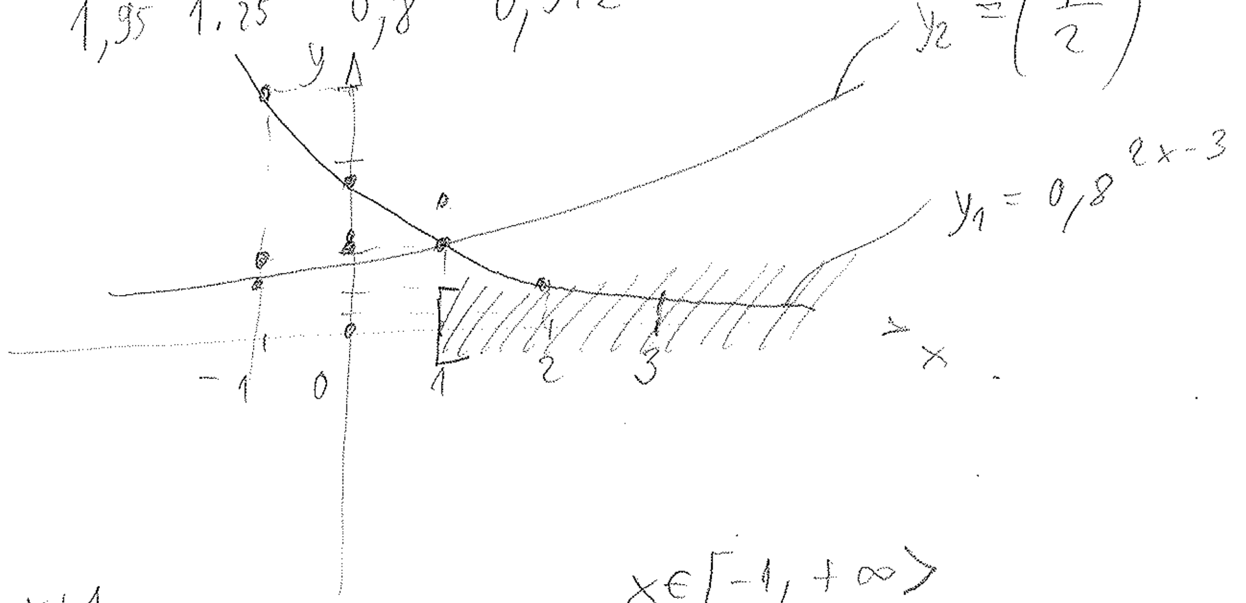
$$2x + \frac{1}{2}x \geq -\frac{1}{2} + 3$$

$$\frac{5}{2}x \geq \frac{5}{2}$$

$$x \geq 1$$

$0,8^{2x-3}$

-1	0	1	2	3
$3,05$	$1,95$	1.25	$0,8$	$0,512$



$x \in [-1, +\infty\rangle$

$\left(\frac{\sqrt{5}}{2}\right)^{x+1}$

-1	0	1	2	3
1	$1,12$	$1,25$	$1,4$	$1,56$

10 (Primjer 19)
Riješimo nejednadžbu: $0,4^{x} - 2,5^{x+1} > 1,5$

$$\left(\frac{4}{10}\right)^{x} - \left(\frac{25}{10}\right)^{x+1} > \frac{15}{10}$$

$$\left(\frac{2}{5}\right)^{x} - \left(\frac{5}{2}\right)^{x+1} > \frac{3}{2}$$

supstitucija: $t = \left(\frac{2}{5}\right)^{x}$

$$t - \frac{5}{2t} > \frac{3}{2} \quad / \cdot 2$$

$$2t - \frac{5}{t} > 3 \quad / \cdot t$$

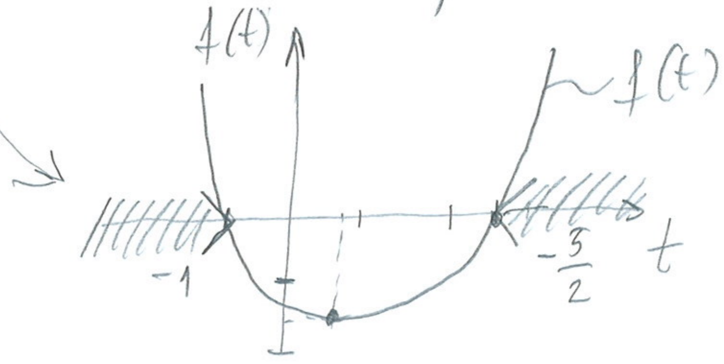
$$2t^2 - 5 > 3t$$

$$2t^2 - 3t - 5 > 0$$

$$t_{1,2} = \frac{3 \pm \sqrt{9 + 40}}{4} = \frac{3 \pm 7}{4} \rightarrow t_1 = -1, \quad t_2 = \frac{5}{2}$$

$t_1 < -1$

$t_2 > \frac{5}{2}$



$$t_0 = -\frac{b}{2a} = \frac{3}{4}$$

$$f(t_0) = 2 \cdot \left(\frac{3}{4}\right)^2 - 3 \cdot \frac{3}{4} - 5$$

$$= \cancel{2}^{1} \cdot \frac{9}{\cancel{16}_{8}} - \frac{9}{4} - 5$$

$$= \frac{9 - 18 - 40}{8} = -\frac{12}{8} = -\frac{3}{2}$$

I) $\left(\frac{2}{5}\right)^x < -1$ $\ominus$ ne može biti negativno rješenje

II) $\left(\frac{2}{5}\right)^x > \left(\frac{2}{5}\right)^{-1} \Rightarrow x < -1$, jer je $\frac{2}{5} < 1$

$0 < a < 1$

DZ

zadaci - 9, 13, 17, 23, 36, 43, 46, 48
ispit 1 - 1, 6, 9
ispit 2 - 4, 8, 9
ispit 3 - 6, 7, 9
ispit 4 - 8, 9, 10
ispit 5 - 7, 10